

1. Each dynamical system below has an equilibrium value of 20. Determine if this equilibrium value is stable or unstable for each system. You do not need to show your work here.

(a)  $u(n) = 0.8u(n-1) + 4$

STABLE

$n$	$u(n)$	$n$	$u(n)$
0	6	0	30
5	15.4125	5	23.2768
10	18.4968	10	21.0737
15	19.5074	15	20.3518
20	19.8386	20	20.1153
25	19.9471	25	20.0378
30	19.9827	30	20.0124
35	19.9942	35	20.0041
40	19.9981	40	20.0013
45	19.9994	45	20.0004
50	19.9998	50	20.0001

(b)  $P(t) = 1.1P(t-1) - 2$

UNSTABLE

$t$	$P(t)$	$t$	$P(t)$
0	10	0	28
3	6.69	3	30.648
6	2.2844	6	34.172
9	-3.579	9	38.864
12	-11.38	12	45.107
15	-21.77	15	53.418
18	-35.6	18	64.479
21	-54.0	21	79.202
24	-78.5	24	98.798

$$(c) v(n+1) = -0.9v(n) + 38$$

STABLE

<u>n</u>	<u>v(n)</u>	<u>n</u>	<u>v(n)</u>
0	15	0	27
5	22.952	5	15.867
10	18.257	10	22.441
15	21.029	15	18.559
20	19.392	20	20.851
25	20.359	25	19.497
30	19.788	30	20.297
35	20.125	35	19.825
40	19.926	40	20.103
45	20.044	45	19.939
50	19.974	50	20.036

$$(d) Q(t+1) = 0.95Q(t) + 1$$

STABLE

<u>t</u>	<u>Q(t)</u>	<u>t</u>	<u>Q(t)</u>
0	16	0	25
10	17.605	10	22.994
20	18.566	20	21.792
30	19.141	30	21.073
40	19.486	40	20.643
50	19.692	50	20.385
60	19.816	60	20.230
70	19.890	70	20.138
80	19.934	80	20.083
90	19.960	90	20.049
100	19.976	100	20.030
110	19.986	110	20.018
120	19.992	120	20.011

2. Find the equilibrium value for the following dynamical system. Show your work.

$$P(t+1) = 0.4P(t) + 18$$

$$P^* = 0.4P^* + 18$$

$$0.6P^* = 18$$

$$P^* = \frac{18}{0.6}$$

$$P^* = 30$$

CHECK ON CALCULATOR

t	P(t)
0	30
1	30
2	30
3	30
⋮	⋮
⋮	⋮



3. Find the equilibrium point for the following dynamical system. Show your work.

$$u(n) = 0.9u(n-1) + 0.8v(n-1) - 14.8$$

$$v(n) = 0.6u(n-1) + 0.8v(n-1) - 3.2$$

$$u^* = 0.9u^* + 0.8v^* - 14.8$$

$$v^* = 0.6u^* + 0.8v^* - 3.2$$

$$10u^* = 9u^* + 8v^* - 148$$

$$10v^* = 6u^* + 8v^* - 32$$

$$u^* = 8v^* - 148$$

$$2v^* = 6u^* - 32$$

$$2v^* = 6(8v^* - 148) - 32$$

$$2v^* = 48v^* - 888 - 32$$

$$920 = 46v^*$$

$$v^* = \frac{920}{46} = 20$$

$$u^* = 8(20) - 148 = 12$$

Equil. value is  
 $(u^*, v^*) =$   
 $(12, 20)$

CHECK ON  
 CALCULATOR

n	u(n)	v(n)
0	12	20
1	12	20
2	12	20
3	12	20
⋮	⋮	⋮
⋮	⋮	⋮



4. Suppose a doctor prescribes that a patient take an initial dose of 30 milligrams of some drug following by a daily maintenance dose of 45 milligrams. The doctor knows that the level of the drug in the patient's bloodstream will increase over time and approach the target goal of 120 milligrams. What percentage of the drug is eliminated each day from the the patient's bloodstream?

LET  $d(n)$  REPRESENT THE # mg OF THIS DRUG IN BLOODSTREAM  $n$  DAYS AFTER THE INITIAL DOSE.

$$d(n) = d(n-1) - pd(n-1) + 45$$

$$d(n) = (1-p)d(n-1) + 45$$

$$d^* = (1-p)d^* + 45$$

$$120 = (1-p) \cdot 120 + 45$$

$$1-p = \frac{75}{120}$$

$$p = 1 - \frac{75}{120}$$

$$p = 0.375$$

37.5%

5. Suppose you borrow \$75,000 at an 9% annual interest rate compounded monthly to be paid back in monthly payments of \$1500.

(a) Write down a discrete dynamical system with initial condition to represent the balance of the loan just after each month's payment.

$$b(n) = b(n-1) + \text{INTEREST} - \text{PAYMENT}$$

$$b(n) = b(n-1) + \frac{0.09}{12} b(n-1) - 1500$$

$$b(n) = 1.0075 b(n-1) - 1500$$

$$b(0) = 75000$$

(b) How many months will it take to pay back the loan?

$n$	$b(n)$
0	75000
⋮	⋮
62	1343.21
63	-146.71

63 months

(c) The last payment will be a bit different than each of the preceding monthly payments. To the nearest penny, what will be the amount of this last payment?

First 62 payments are each \$1500

THE 63rd AND FINAL PAYMENT IS

$$1500 - 146.71 = \$1353.29$$