Math 172 Quiz 3 Spring 2008

Name SOLUTIONS

1. (2 points) A biologist studied the growth of a rabbit population in a field. She let $f(t)$ represent the number of rabbits t weeks from the start of her research. Suppose that $f'(8) = 9$. Which of the following sentences must be true?

   (a) Nine weeks after the start of her research, there were eight rabbits in the field.

   (b) Eight weeks after the start of her research, there were nine rabbits in the field.

   (c) Nine weeks after the start of her research, the rabbit population was increasing by eight rabbits per week.

   (d) Eight weeks after the start of her research, the rabbit population was increasing by nine rabbits per week.

   (e) During the first eight weeks of her research, the rabbit population increased an average of nine rabbits per week.

   (f) During the first nine weeks of her research, the rabbit population increased an average of eight rabbits per week.

2. A model for the population of a town predicts the population t years from now to be given by $P(t) = 750e^{-0.03t}$.

   (a) (1 point) What population does this model predict for this town 20 years from now?

   $P(20) = 750e^{-0.03(20)}$

   ≈ 411.6 people

   (b) (2 points) How quickly in people per year is the population predicted to be changing 20 years from now?

   $P'(t) = 750e^{-0.03t} \cdot (-0.03)$

   $P'(t) = -22.5e^{-0.03t}$

   $P'(20) = -22.5e^{-0.03(20)}$

   ≈ -12.3 people/year

3. (2 points) Find the equation of the line tangent to the graph of $y = x^3 - 16x$ at $x = 5$.

WHEN $x=5$, $y = 5^3 - 16(5) = 45$

SO WE HAVE THE POINT $(5,45)$

SINCE $y' = 3x^2 - 16$, AT $x=5$ WE GET $y' = 3(5)^2 - 16 = 59$, SO SLOPE $= 59$

THUS $\boxed{y - 45 = 59(x-5)}$ or $\boxed{y = 59x - 250}$ ☆

4. (3 points) Find derivatives of the following functions. Use Leibniz notation for the derivative.

(a) $y = \ln\left(3x^5 + 10x^2 + 25\right)$

$$\frac{dy}{dx} = \frac{1}{3x^5 + 10x^2 + 25} \cdot (15x^4 + 20x)$$

$$\frac{dy}{dx} = \frac{15x^4 + 20x}{3x^5 + 10x^2 + 25}$$

(b) $h = \dfrac{6}{r^3}$

$$h = 6r^{-3}$$

$$\frac{dh}{dr} = -18r^{-4} = \frac{-18}{r^4}$$

(c) $P = 5t^2e^{-t}$

$$\frac{dP}{dt} = (5t^2)'(e^{-t}) + (5t^2)(e^{-t})'$$

$$= 10te^{-t} + 5t^2e^{-t} \cdot (-1)$$

$$\frac{dP}{dt} = 10te^{-t} - 5t^2e^{-t}$$