

1. Find the equilibrium values, if any exist, for each of the following discrete dynamical systems.

(a)  $P(t) = 0.5P(t - 1) + 3$

(b)  $u(n + 1) = -u(n) + 8$

(c)  $h(t) = h(t - 1) + 2$

(d)  $u(n) = u(n - 1)$

2. Suppose that the equilibrium value is 5 for the dynamical system  $P(t) = 1.3P(t - 1) + b$ . What is the value of  $b$ ?

3. Find the equilibrium point, if it exists, for the following dynamical system.

$$P(t + 1) = 1.1P(t) + 3Q(t) - 2$$

$$Q(t + 1) = 2Q(t)$$

4. Find the equilibrium point, if it exists, for the following dynamical system.

$$u(n) = 0.6u(n - 1) + 1.2v(n - 1) + 18$$

$$v(n) = 0.2u(n - 1) + 0.4v(n - 1) + 1$$

5. Find the equilibrium value for the following dynamical system. Is the equilibrium value stable or unstable?

$$u(n) = 0.9u(n - 1) - 3.5$$

6. Find the equilibrium value for the following dynamical system. Is the equilibrium value stable or unstable?

$$u(n) = 1.25u(n - 1) - 6.1$$

7. Find the equilibrium point, if it exists, for the following dynamical system. Is the equilibrium point stable or unstable?

$$u(n) = 2u(n - 1) + v(n - 1) + 3$$

$$v(n) = 4u(n - 1) - v(n - 1) + 6$$

8. Find the equilibrium point, if it exists, for the following dynamical system. Is the equilibrium point stable or unstable?

$$u(n) = 0.8u(n-1) - v(n-1) + 7$$

$$v(n) = u(n-1) + 1.2v(n-1) - 11$$

9. Find the equilibrium point, if it exists, for the following dynamical system. Is the equilibrium point stable or unstable?

$$u(n) = 0.3u(n-1) - 0.5v(n-1) + 30$$

$$v(n) = 0.2u(n-1) + v(n-1) - 4$$

10. A doctor prescribes some drug to be taken every 4 hours. Suppose the body eliminates 30% of this drug every 4 hours. What should the prescribed dosage of this drug be if the doctor wants the equilibrium value (the target goal) to be equal to 120 *ml* ?
11. Suppose a patient takes 40 milligrams of a medicine every day. Through a blood test, it is determined that the amount of this drug in the patient's blood has stabilized at 140 milligrams. What percentage of the drug does this patient's kidneys eliminate each day?
12. Assume that

- there are 2 chemicals in the body,  $U$  and  $V$
- the body filters out 10% of  $U$  and 15% of  $V$  each day through the kidneys
- liver enzymes metabolize 40% of  $U$  into  $V$  and 30% of  $V$  into  $U$  each day

In addition we assume that no  $U$  is ingested but each day  $x$  mg of  $V$  are taken.

- (a) What should  $x$  be so that the equilibrium value for  $V$  is 200 mg ?
- (b) What will be the resulting equilibrium value for  $U$  given the prescribed dosage of  $V$  found in part (a) ?
13. Vitamin A is stored primarily in our plasma and liver. Suppose that 40% of the vitamin A in the plasma is filtered out by the kidneys each day and that 30% of the vitamin A in the plasma is absorbed into the liver each day. Also assume that 1% of the vitamin A in the liver is absorbed back into the plasma each day. Suppose you have a daily intake of 1 mg of vitamin A each day, which goes directly into the plasma.
- (a) Determine discrete dynamical systems for  $P(t)$  and  $L(t)$ , the number of milligrams of vitamin A in the plasma and liver, respectively,  $t$  days after your first dose of vitamin A.
- (b) Find the equilibrium amounts of vitamin A in the plasma and liver.

14. Maria makes a single \$1000 purchase on her credit card and then never uses it again. The card carries an 18% annual interest rate compounded monthly, and she only pays back her minimum monthly payment of \$20 per month.
- (a) Write down a discrete dynamical system with initial condition to represent the amount Maria owes after  $n$  monthly payments.
  - (b) How many months will it take Maria to pay off this credit card debt?
  - (c) Explain what happens if Maria's single purchase was for \$1500 instead of \$1000.
15. Suppose you borrow \$120,000 at an 8.4% annual interest rate compounded monthly to be paid back in monthly payments of \$1800.
- (a) Write down a discrete dynamical system with initial condition to represent the balance of the loan just after each month's payment.
  - (b) How many months will it take to pay back the loan?
  - (c) The last payment will be a bit different than each of the preceding monthly payments. To the nearest penny, what will be the amount of this last payment?
16. Suppose you have a college loan of \$30000 when you graduate from college. This loan has a 6% annual interest rate, compounded monthly. You must pay this loan back in 120 equal monthly payments. What are those payments, rounding the answer up to the nearest cent? What is the last payment, given that you don't overpay this loan?
17. Suppose you wish to buy a house but can only afford \$1100 a month. Your loan will carry an 8.1% annual interest rate compounded monthly.
- (a) To the nearest thousand dollars, how large can your loan be if it is to be paid off in 20 years?
  - (b) To the nearest thousand dollars, how large can your loan be if it is to be paid off in 30 years?