Math 172 **Worksheet C Solutions** **Spring 2008**

1. Suppose $p(n)$ represents some population n years from now, and that this population is modeled by the following discrete dynamical system.

$$p(n) = p(n-1) + 40$$
$$p(0) = 60$$

Which one of the following statements follows from this model?

(a) The population will increase by 60% per year.
(b) The population will decrease by 60% per year.
(c) The population will increase by 60 people per year.
(d) The population will decrease by 60 people per year.
(e) The population will increase by 40% per year.
(f) The population will decrease by 40% per year.
(g) The population will increase by 40 people per year.
(h) The population will decrease by 40 people per year.

2. Suppose $a(n)$ represents the number of milligrams of some drug in the bloodstream n hours from now, and that the amount of this drug in the bloodstream is modeled by the following discrete dynamical system.

$$a(n) = 0.75a(n-1)$$
$$a(0) = 50$$

Which one of the following statements follows from this model?

~~(a) The amount of drug in the bloodstream will increase by 25% per hour.~~
(b) The amount of drug in the bloodstream will decrease by 25% per hour.
(c) The amount of drug in the bloodstream will increase by 25 milligrams per hour.
(d) The amount of drug in the bloodstream will decrease by 25 milligrams per hour.
(e) The amount of drug in the bloodstream will increase by 50% per hour.
(f) The amount of drug in the bloodstream will decrease by 50% per hour.
(g) The amount of drug in the bloodstream will increase by 50 milligrams per hour.
(h) The amount of drug in the bloodstream will decrease by 50 milligrams per hour.
(i) The amount of drug in the bloodstream will increase by 75% per hour.
(j) The amount of drug in the bloodstream will decrease by 75% per hour.
(k) The amount of drug in the bloodstream will increase by 75 milligrams per hour.
(l) The amount of drug in the bloodstream will decrease by 75 milligrams per hour.

3. Determine the equilibrium value, if it exists, for each of the following discrete dynamical systems. Is the equilibrium value stable or unstable? If it has an equilibrium value, then by what percentage does the function get closer to or further away from its equilibrium value each time period?

(a) $u(n) = u(n-1) - 5$ and $u(0) = 40$

NO EQUILIBRIUM VALUE SINCE u IS A LINEAR FUNCTION WITH NONZERO SLOPE.

ALSO $u^* = u^* - 5 \Rightarrow 0 = -5$ WHICH IS FALSE.

(b) $P(t+1) = P(t) + 3$ and $P(0) = 100$

NO EQUILIBRIUM VALUE SINCE P IS A LINEAR FUNCTION WITH NONZERO SLOPE.

ALSO $P^* = P^* + 3 \Rightarrow 0 = 3$ WHICH IS FALSE.

(c) $u(n+1) = 1.2u(n)$ and $u(0) = 60$

$u^* = 1.2u^* \Rightarrow$ $u^* = 0$ IS THE ONLY EQUILIBRIUM VALUE.

SINCE $1.2 > 1$, $u^* = 0$ IS AN UNSTABLE EQUILIBRIUM VALUE

$$\lim_{n\to\infty} \frac{u(n)-0}{u(n-1)-0} = \lim_{n\to\infty} \frac{u(n)}{u(n-1)} = \lim_{n\to\infty} \frac{1.2u(n-1)}{u(n-1)} = 1.2$$

SO THE FUNCTION u MOVES 20% FURTHER AWAY FROM EQUILIBRIUM EACH TIME PERIOD

(d) $Q(t) = 0.87Q(t-1)$ and $Q(0) = 150$

$Q^* = 0.87Q^* \Rightarrow$ $Q^* = 0$ IS THE EQUILIBRIUM VALUE

SINCE $-1 < 0.87 < 1$, $Q^* = 0$ IS A STABLE EQUIL. VALUE

$$\lim_{t\to\infty} \frac{Q(t)-0}{Q(t-1)-0} = \lim_{t\to\infty} \frac{Q(t)}{Q(t-1)} = \lim_{t\to\infty} \frac{0.87Q(t-1)}{Q(t-1)} = 0.87$$

SO THE FUNCTION Q MOVES 13% CLOSER TO EQUILIBRIUM EACH TIME PERIOD

(e) $W(n+1) = 1.1W(n) - 5$ and $W(0) = 40$

$W^* = 1.1W^* - 5$

$5 = 0.1W^*$

$W^* = 50$ IS THE EQUIL. VALUE

SINCE $1.1 > 1$, $W^* = 50$ IS AN UNSTABLE EQUIL. VALUE

$$\lim_{n\to\infty} \frac{W(n)-50}{W(n-1)-50} = \lim_{n\to\infty} \frac{(1.1W(n-1)-5)-50}{W(n-1)-50}$$

$$= \lim_{n\to\infty} \frac{1.1(W(n-1)-50)}{W(n-1)-50} = 1.1$$

SO THE FUNCTION W MOVES 10% FURTHER AWAY FROM EQUILIBRIUM EACH TIME PERIOD.

(f) $P(t) = 0.95P(t-1) + 5$ and $P(0) = 20$

$P^* = 0.95P^* + 5$

$0.05P^* = 5$

$P^* = \frac{5}{0.05} = 100$ IS THE EQUIL. VALUE

SINCE $-1 < 0.95 < 1$, $P^* = 100$ IS A STABLE EQUIL. VALUE

$$\lim_{t\to\infty} \frac{P(t)-100}{P(t-1)-100} = \lim_{t\to\infty} \frac{(0.95P(t-1)+5)-100}{P(t-1)-100}$$

$$= \lim_{t\to\infty} \frac{0.95(P(t-1)-100)}{P(t-1)-100} = 0.95$$

SO THE FUNCTION P MOVES 5% CLOSER TO EQUILIBRIUM EACH TIME PERIOD.

4. Find an explicit formula for each function in the preceding problem.

(3a) $u(n) = -5n + 40$

(3b) $P(t) = 3t + 100$

(3c) $u(n) = 60(1.2)^n$

(3d) $Q(t) = 150(0.87)^t$

([redacted])

(3e) $W(n) = K(1.1)^n + 50$

EQ. VALUE

$W(0) = 40$ SO WE GET

$40 = K(1.1)^0 + 50$

$K = -10$

$W(n) = -10(1.1)^n + 50$

(3f) $P(t) = K(0.95)^t + 100$

EQ. VALUE

$P(0) = 20$ SO WE GET

$20 = K(0.95)^0 + 100$

$K = -80$

$P(t) = -80(0.95)^t + 100$