1. Suppose that y is a function of x which satisfies the differential equation

$$\frac{dy}{dx} = 3(x-2)(x-4)$$

   (a) Specify an interval of either x-values or y-values for which the graph of y will be increasing.
   (b) Specify an interval of either x-values or y-values for which the graph of y will be decreasing.

2. Suppose that y is a function of x which satisfies the differential equation

$$\frac{dy}{dx} = 3(y-2)(y-4)$$

   (a) Specify an interval of either x-values or y-values for which the graph of y will be increasing.
   (b) Specify an interval of either x-values or y-values for which the graph of y will be decreasing.

3. Consider the population model for logistic growth

$$\frac{dP}{dt} = 0.15P\left(1 - \frac{P}{6000}\right)$$

   where P is the population at time t.

   (a) For which values of P is the population increasing?
   (b) For which values of P is the population decreasing?
   (c) For which values of P is the population in equilibrium? Determine whether each of these equilibrium values is stable or unstable.
   (d) Sketch a rough graph of P given that: (i) $P(0) = 500$, (ii) $P(0) = 4000$, (iii) $P(0) = 6000$, (iv) $P(0) = 8000$.

4. Consider the population model

$$\frac{dP}{dt} = 0.3P\left(1 - \frac{P}{200}\right)\left(\frac{P}{50} - 1\right)$$

   where P is the population at time t.

   (a) For which values of P is the population increasing?
   (b) For which values of P is the population decreasing?
   (c) For which values of P is the population in equilibrium? Determine whether each of these equilibrium values is stable or unstable.
   (d) Sketch a rough graph of P given that: (i) $P(0) = 10$, (ii) $P(0) = 50$, (iii) $P(0) = 150$, (iv) $P(0) = 300$.

5. Find all equilibrium values for the following differential equation. There is no need to discuss whether or not these equilibrium values are stable.

$$\frac{dP}{dt} = 0.5(P-4)(P+3)(P^2-25)$$

6. Suppose y is a function of t which satisfies the differential equation below.

$$\frac{dy}{dt} = 0.25(y-10)(20-y)$$

Sketch plausible graphs for y as a function of t given each initial value below. Your graphs should clearly show if the y-values approach any particular values (i.e. horizontal asymptotes). You should draw all five graphs together on one set of coordinate axes.

(a) $y(0) = 25$

(b) $y(0) = 20$

(c) $y(0) = 15$

(d) $y(0) = 10$

(e) $y(0) = 5$

3 Important Growth Models

- LINEAR GROWTH (m = slope, P_0 = initial value)

Type	Linear Model	Explicit Solution
continuous	$\frac{dP}{dt} = m$	$P = mt + P_0$
discrete	$P(t) = P(t-1) + m$	$P = mt + P_0$

- EXPONENTIAL GROWTH (r = growth rate, P_0 = initial value)

Type	Exponential Model	Explicit Solution
continuous	$\frac{dP}{dt} = rP$	$P = P_0 e^{rt}$
discrete	$P(t) = P(t-1) + rP(t-1)$	$P = P_0\,(1+r)^n$

- LOGISTIC GROWTH (r = growth rate, k = carrying capacity, P_0 = initial value)

Type	Logistic Model
continuous	$\frac{dP}{dt} = rP\left(1 - \frac{P}{k}\right)$
discrete	$P(t) = P(t-1) + rP(t-1)\left(1 - \frac{P(t-1)}{k}\right)$

7. If P represents some population t years from now, and $\frac{dP}{dt} = 20$, then which of the following statements is correct?

   (a) P grows linearly by 20 people per year.
   (b) P grows linearly by 120 people per year.
   (c) P grows exponentially by 20% per year.
   (d) P grows logistically with a carrying capacity of 20.

8. If P represents some population t years from now, and $P(t) = 1.2P(t-1)$, then which of the following statements is correct?

   (a) P grows linearly by 20 people per year.
   (b) P grows linearly by 120 people per year.
   (c) P grows exponentially by 20% per year.
   (d) P grows logistically with a carrying capacity of 120.

9. The population of a city was 5000 in 1980. Since then the population has been increasing by 100 people per year.

   (a) Determine a discrete dynamical system with initial value to model the city's population.
   (b) Determine a differential equation with initial value to model the city's population.
   (c) Determine an explicit formula for the city's population.
   (d) What does your model predict for the city's population in the year 2000?
   (e) When does your model predict the population will have reached 10000?

10. An initial deposit of $200 is made into an account with an annual interest rate of 3% compounded annually.

    (a) Determine a discrete dynamical system with initial value to model the amount of money in this account.
    (b) Determine an explicit formula for the amount of money in this account.
    (c) How much money will the account hold 8 years after the initial deposit?
    (d) How long will it take until the balance in this account is $500?

11. An initial deposit of 200 is made into an account with an annual interest rate of 3% compounded continuously.

    (a) Determine a differential equation with initial value to model the amount of money in this account.
    (b) Determine an explicit formula for the amount of money in this account.
    (c) How much money will the account hold 8 years after the initial deposit?
    (d) How long will it take until the balance in this account is $500?

12. Why did I suggest using a discrete dynamical system for problem (10), but a differential equation for problem (11)?

13. The number of fish in a pond was 300 in 1970. Since then the number of fish has been increasing by 5% per year.

    (a) Determine a discrete dynamical system with initial value to model the number of fish in this pond.

    (b) Enter this system into your calculator to make a table of values for the number of fish in the pond each year from 1970 to 1980.

    (c) Determine an explicit formula for the number of fish in the pond.

    (d) When will the fish population reach 750?

14. The number of fish in a pond was 300 in 1970. Since then the number of fish has been increasing by 5% per year.

    (a) Determine a differential equation with initial value to model the number of fish in this pond.

    (b) Use Euler's Method with $\Delta t = 1$ to make a table of values for the number of fish in the pond each year from 1970 to 1980.

    (c) Determine an explicit formula for the number of fish in the pond.

    (d) When will the fish population reach 750?

15. Which model was best to use for the fish population – the discrete dynamical system or the differential equation? Why?

16. There are currently 5000 deer in a forest. Suppose the population of deer grows logistically with an intrinsic growth rate of 6% and a carrying capacity of $20,000$.

    (a) Sketch a rough graph of the deer population.

    (b) Determine a discrete dynamical system with initial value to model the deer population.

    (c) Make a table of values for the number of deer your discrete model predicts for the next 4 years.

    (d) Determine a differential equation with initial value to model the deer population.

    (e) Use Euler's Method with $\Delta t = 1$ to make a table of values for the number of deer your continuous model predicts for the next 4 years.

    (f) Use Euler's Method with $\Delta t = 0.5$ to make a table of values for the number of deer your continous model predicts for the next 4 years.

    (g) How close are the approximations for discrete and continuous models?