Math 172 (Section 1) Test 2 Spring 2009

Name SOLUTIONS

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For multiple-choice questions, precisely one answer is correct. Circle this correct answer.
- For all other questions, you must show sufficient work to justify your answer.
- You are not allowed to borrow another student's calculator during the test.

Do not write below this line

#1 (6 points)	______	#6 (5 points)	______
#2 (6 points)	______	#7 (8 points)	______
#3 (6 points)	______	#8 (8 points)	______
#4 (10 points)	______	#9 (9 points)	______
#5a (6 points)	______	#10 (6 points)	______
#5b (6 points)	______		
#5c (6 points)	______		
#5d (6 points)	______		
#5e (6 points)	______		
#5f (6 points)	______		

TOTAL (100 points) ______

1. (6 points) Let P represent some population t years from now. Which one of the following statements is correct given that $P(0) = 70$ and $\frac{dP}{dt} = 20$?

   (a) P grows linearly by 20 people per year.

   (b) P grows linearly by 30 people per year.

   (c) P grows linearly by 70 people per year.

   (d) P grows linearly by 80 people per year.

   (e) P grows exponentially with a continuous growth rate of 20% per year.

   (f) P grows exponentially with a continuous growth rate of 30% per year.

   (g) P grows exponentially with a continuous growth rate of 70% per year.

   (h) P grows exponentially with a continuous growth rate of 80% per year.

   (i) P grows logistically with a carrying capacity of 20.

   (j) P grows logistically with a carrying capacity of 30.

   (k) P grows logistically with a carrying capacity of 70.

   (l) P grows logistically with a carrying capacity of 80.

2. (6 points) Let P represent some population t years from now. Which one of the following statements is correct given that $P(0) = 70$ and $\frac{dP}{dt} = 0.2P$?

   (a) P grows linearly by 20 people per year.

   (b) P grows linearly by 30 people per year.

   (c) P grows linearly by 70 people per year.

   (d) P grows linearly by 80 people per year.

   (e) P grows exponentially with a continuous growth rate of 20% per year.

   (f) P grows exponentially with a continuous growth rate of 30% per year.

   (g) P grows exponentially with a continuous growth rate of 70% per year.

   (h) P grows exponentially with a continuous growth rate of 80% per year.

   (i) P grows logistically with a carrying capacity of 20.

   (j) P grows logistically with a carrying capacity of 30.

   (k) P grows logistically with a carrying capacity of 70.

   (l) P grows logistically with a carrying capacity of 80.

3. (6 points) A model for the population (in thousands) of a city predicts the population t years from now to be given by $P(t) = 300e^{0.05t}$. In 12 years this model predicts that population will be increasing by approximately

$P'(t) = 300e^{0.05t}(0.05)$

$P'(t) = 15e^{0.05t}$

$P'(12) \approx 27.332$ thousand

(a) 28155 people per year.

(b) 27332 people per year.

(c) 26531 people per year.

(d) 25708 people per year.

(e) 24822 people per year.

(f) 23976 people per year.

4. (10 points) Suppose P is a function of t whose growth is determined by the differential equation with initial condition shown. Use Euler's Method with $\Delta t = 3$ to approximate $P(9)$. Each step in your calculation should be correctly rounded off to three places after the decimal point.

$$\frac{dP}{dt} = \frac{5}{P^2} \quad \text{and} \quad P(0) = 3$$

t	P	dP/dt	CALCULATIONS
0	3	0.556	$3 + (0.556)(3)$
3	4.667	0.230	$4.667 + (0.230)(3)$
6	5.355	0.174	$5.355 + (0.174)(3)$
9	5.878		

$P(9) \approx 5.878$

5. (6 points each) Let P represent a town's population t years from now. Give a differential equation which models the population under the following conditions. You do not need to include an initial value.

(a) The population is increasing at a rate of 35 people per year.

$$\frac{dP}{dt} = 35$$

(b) The population is decreasing at a rate of 40 people per year.

$$\frac{dP}{dt} = -40$$

(c) The population is increasing at a continuous rate of 7% per year.

$$\frac{dP}{dt} = 0.07P$$

(d) The population is decreasing at a continuous rate of 4% per year.

$$\frac{dP}{dt} = -0.04P$$

(e) The population is growing at a rate which is proportional to the population size with a constant of proportionality of 0.02.

$$\frac{dP}{dt} = 0.02P$$

(f) The population is growing logistically with an intrinsic growth rate of 4% per year and a carrying capacity of 600.

$$\frac{dP}{dt} = 0.04P\left(1 - \frac{P}{600}\right)$$

6. (5 points) Find all equilibrium values for the following differential equation. There is no need to discuss whether or not these equilibrium values are stable.

$$\frac{dP}{dt} = 0.8(P^2+9)(P^2-25)(2P-5)(P-7)(P-2)^3$$

$$0 = 0.8(P^2+9)(P^2-25)(2P-5)(P-7)(P-2)^3$$

no equil values

$P = \pm 5$ $\quad P = 2.5$ $\quad P = 7$ $\quad P = 2$

equil. values are $5, -5, 2.5, 7, 2$

7. (8 points) Given that $\frac{dP}{dt} = 8$ and $P(0) = 150$, find an explicit formula for P.

$$P = 8t + 150$$

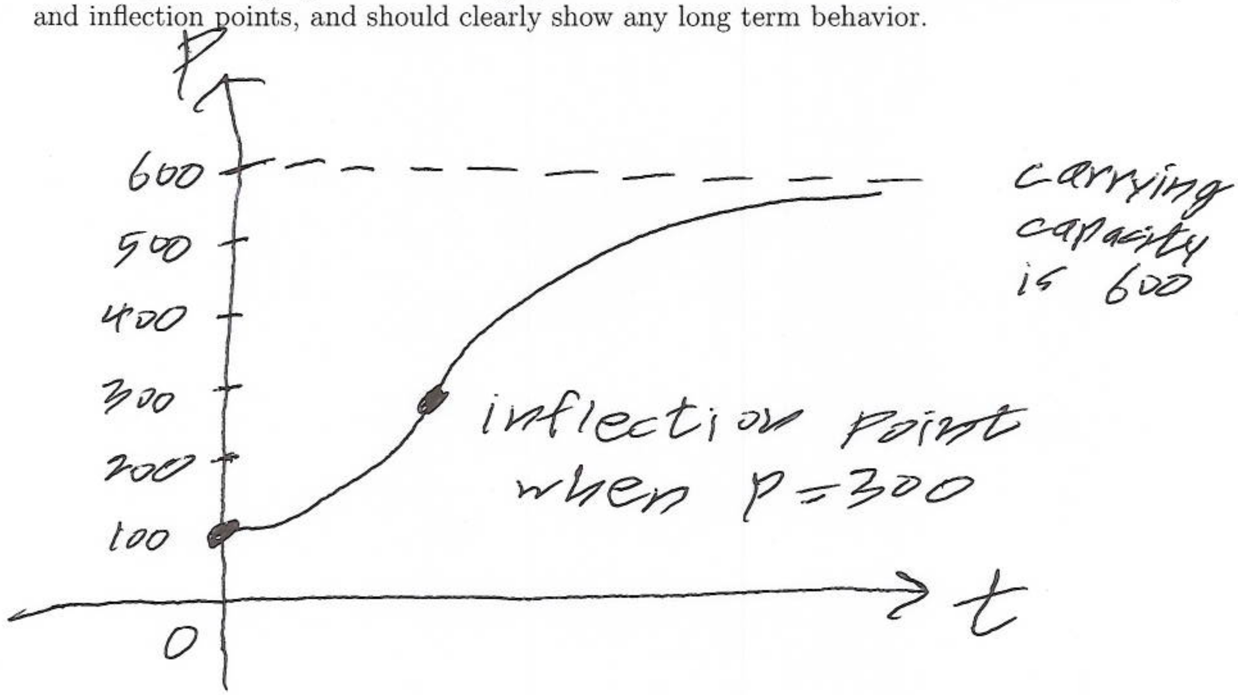
8. (8 points) Given that $\frac{dP}{dt} = 0.25P$ and $P(0) = 100$, find an explicit formula for P.

$$P = 100e^{0.25t}$$

9. (9 points) Let P represent a town's population t years from now. The current population is 100 and the following model predicts its growth.

$$\frac{dP}{dt} = 0.035P\left(1 - \frac{P}{600}\right)$$

Sketch a plausible graph for P. Your graph should include all known coordinates for intercepts and inflection points, and should clearly show any long term behavior.

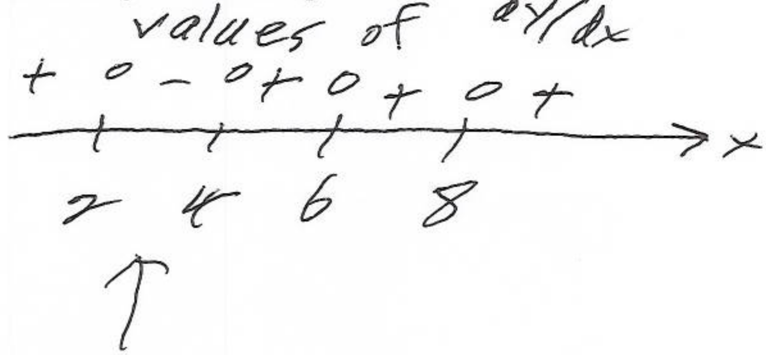


10. (6 points) Suppose that y is a function of x which satisfies the differential equation

$$\frac{dy}{dx} = (y-2)(y-4)(y-6)^2(y-8)^2$$

Upon which one of the following intervals must y be decreasing?

(a) $y \in (-\infty, 2]$
(b) $y \in [0, 2]$
(c) $y \in [2, 4]$
(d) $y \in [4, 6]$
(e) $y \in [6, 8]$
(f) $y \in [8, \infty)$



y decreasing since $\frac{dy}{dx} < 0$