

Name _____

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For multiple-choice questions, precisely one answer is correct. Circle this correct answer.
- For all other questions, you must show sufficient work to justify your answer.
- You are not allowed to borrow another student's calculator during the test.

Do not write below this line

#1 (6 points) _____

#6 (5 points) _____

#2 (6 points) _____

#7 (8 points) _____

#3 (6 points) _____

#8 (8 points) _____

#4 (10 points) _____

#9 (9 points) _____

#5a (6 points) _____

#10 (6 points) _____

#5b (6 points) _____

#5c (6 points) _____

#5d (6 points) _____

#5e (6 points) _____

#5f (6 points) _____

TOTAL (100 points) _____

1. (6 points) Let P represent some population t years from now. Which one of the following statements is correct given that $P(0) = 70$ and $\frac{dP}{dt} = 20$?
- (a) P grows linearly by 20 people per year.
 - (b) P grows linearly by 30 people per year.
 - (c) P grows linearly by 70 people per year.
 - (d) P grows linearly by 80 people per year.
 - (e) P grows exponentially with a continuous growth rate of 20% per year.
 - (f) P grows exponentially with a continuous growth rate of 30% per year.
 - (g) P grows exponentially with a continuous growth rate of 70% per year.
 - (h) P grows exponentially with a continuous growth rate of 80% per year.
 - (i) P grows logistically with a carrying capacity of 20.
 - (j) P grows logistically with a carrying capacity of 30.
 - (k) P grows logistically with a carrying capacity of 70.
 - (l) P grows logistically with a carrying capacity of 80.
2. (6 points) Let P represent some population t years from now. Which one of the following statements is correct given that $P(0) = 70$ and $\frac{dP}{dt} = 0.2P$?
- (a) P grows linearly by 20 people per year.
 - (b) P grows linearly by 30 people per year.
 - (c) P grows linearly by 70 people per year.
 - (d) P grows linearly by 80 people per year.
 - (e) P grows exponentially with a continuous growth rate of 20% per year.
 - (f) P grows exponentially with a continuous growth rate of 30% per year.
 - (g) P grows exponentially with a continuous growth rate of 70% per year.
 - (h) P grows exponentially with a continuous growth rate of 80% per year.
 - (i) P grows logistically with a carrying capacity of 20.
 - (j) P grows logistically with a carrying capacity of 30.
 - (k) P grows logistically with a carrying capacity of 70.
 - (l) P grows logistically with a carrying capacity of 80.

3. (6 points) A model for the population (in thousands) of a city predicts the population t years from now to be given by $P(t) = 300e^{0.05t}$. In 12 years this model predicts that population will be increasing by approximately
- (a) 28155 people per year.
 - (b) 27332 people per year.
 - (c) 26531 people per year.
 - (d) 25708 people per year.
 - (e) 24822 people per year.
 - (f) 23976 people per year.
4. (10 points) Suppose P is a function of t whose growth is determined by the differential equation with initial condition shown. Use Euler's Method with $\Delta t = 3$ to approximate $P(9)$. Each step in your calculation should be correctly rounded off to three places after the decimal point.

$$\frac{dP}{dt} = \frac{5}{P^2} \quad \text{and} \quad P(0) = 3$$

5. (6 points each) Let P represent a town's population t years from now. Give a differential equation which models the population under the following conditions. You do not need to include an initial value.
- (a) The population is increasing at a rate of 35 people per year.

 - (b) The population is decreasing at a rate of 40 people per year.

 - (c) The population is increasing at a continuous rate of 7% per year.

 - (d) The population is decreasing at a continuous rate of 4% per year.

 - (e) The population is growing at a rate which is proportional to the population size with a constant of proportionality of 0.02.

 - (f) The population is growing logistically with an intrinsic growth rate of 4% per year and a carrying capacity of 600.

6. (5 points) Find all equilibrium values for the following differential equation. There is no need to discuss whether or not these equilibrium values are stable.

$$\frac{dP}{dt} = 0.8(P^2 + 9)(P^2 - 25)(2P - 5)(P - 7)(P - 2)^3$$

7. (8 points) Given that $\frac{dP}{dt} = 8$ and $P(0) = 150$, find an explicit formula for P .

8. (8 points) Given that $\frac{dP}{dt} = 0.25P$ and $P(0) = 100$, find an explicit formula for P .

9. (9 points) Let P represent a town's population t years from now. The current population is 100 and the following model predicts its growth.

$$\frac{dP}{dt} = 0.035P \left(1 - \frac{P}{600} \right)$$

Sketch a plausible graph for P . Your graph should include all known coordinates for intercepts and inflection points, and should clearly show any long term behavior.

10. (6 points) Suppose that y is a function of x which satisfies the differential equation

$$\frac{dy}{dx} = (y - 2)(y - 4)(y - 6)^2(y - 8)^2$$

Upon which one of the following intervals must y be decreasing?

- (a) $y \in (-\infty, 2]$
- (b) $y \in [0, 2]$
- (c) $y \in [2, 4]$
- (d) $y \in [4, 6]$
- (e) $y \in [6, 8]$
- (f) $y \in [8, \infty)$