Math 172 (Section 1) Test 3 Spring 2009

Name SOLUTIONS

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- Show sufficient work to justify each answer.
- You are not allowed to borrow another student's calculator during the test.

Do not write below this line

#1 (5 points)	_____	#7 (16 points)	_____
#2 (6 points)	_____	#8 (15 points)	_____
#3 (6 points)	_____	#9 (12 points)	_____
#4 (6 points)	_____	#10 (16 points)	_____
#5 (6 points)	_____		
#6 (12 points)	_____		
		TOTAL (100 points)	_____

Test 1 _____ Test 2 _____ Test 3 _____ Total _____

If you skip the final exam, your course grade will be _____

1. (5 points) Find an explicit solution to the initial value problem.

$$\frac{dP}{dt} = 5 \qquad \text{and} \qquad P(3) = 45$$

$P = 5t + C$

$45 = 5 \cdot 3 + C$

$C = 30$

$P = 5t + 30$

2. (6 points) Find an explicit solution to the initial value problem.

$$\frac{dw}{dt} = 3w \qquad \text{and} \qquad w(0) = 20$$

$\int \frac{dw}{w} = \int 3\,dt$

$\ln w = 3t + C$

$\ln(20) = 3 \cdot 0 + C$

$C = \ln 20$

$\ln w = 3t + \ln 20$

$w = e^{3t + \ln 20}$

$w = e^{3t} \cdot e^{\ln 20}$

$w = 20e^{3t}$

or just memorize

3. (6 points) Find an explicit solution to the initial value problem.

$$\frac{dw}{dt} = 4t \qquad \text{and} \qquad w(0) = 30$$

$w = 2t^2 + C$

$30 = 2 \cdot 0^2 + C$

$C = 30$

$w = 2t^2 + 30$

4. (6 points) Find an explicit solution to the initial value problem.

$$\frac{dy}{dx} = -8e^{-4x} \quad \text{and} \quad y(0) = 10$$

$y = 2e^{-4x} + C$

$10 = 2e^{-4\cdot 0} + C$

$10 = 2 + C$

$C = 8$

$y = 2e^{-4x} + 8$

5. (6 points) Find an explicit solution to the initial value problem.

$$\frac{dy}{dx} = \frac{e^x + 2x}{y^2} \quad \text{and} \quad y(0) = 3$$

$\int y^2\,dy = \int (e^x + 2x)\,dx$

$\frac{1}{3}y^3 = e^x + x^2 + C$

$\frac{1}{3}(3)^3 = e^0 + 0^2 + C$

$9 = 1 + C$

$C = 8$

$\frac{1}{3}y^3 = e^x + x^2 + 8$

$y^3 = 3e^x + 3x^2 + 24$

$y = \sqrt[3]{3e^x + 3x^2 + 24}$

6. (12 points) Suppose that a fish population grows logistically with an intrinsic growth rate of 30% and a carrying capacity of 600.

(a) Determine a discrete dynamical system to model this fish population.

$$P(t) = P(t-1) + 0.3P(t-1)\left(1 - \frac{P(t-1)}{600}\right)$$

(b) Determine the maximum interval of stability for this fish population.

0 is an unstable equilibrium value.
600 is a stable equilibrium value.
The maximum interval of stability for $P^* = 600$ is $(0, 2600)$

You can find this by trying different starting values on the calculator or by solving

$$0 = P + 0.3P\left(1 - \frac{P}{600}\right)$$

to get $P = 0$ or $P = 2600$

7. (16 points) For Nancy's metabolism, the dynamical system modeling the elimination of alcohol is given by

$$a(n) = a(n-1) - \frac{9.5a(n-1)}{4 + a(n-1)} + d$$

where $a(n)$ is the amount of alcohol (in grams) in her bloodstream after n hours of drinking d grams of alcohol per hour.

(a) How many grams of alcohol per hour can Nancy drink if at the end of a 4 hour party she is to have 40 grams of alcohol in her bloodstream? Begin with $a(0) = 0$ and give your answer correct to one place after the decimal.

try different values for d on the calculator to eventually obtain $d = 16.05$ gives $a(4) \approx 39.973$ and $d = 16.06$ gives $a(4) \approx 40.011$

so $16.05 < d < 16.06$

Thus $d \approx 16.1$ grams

(b) Compute the equilibrium amount of alcohol in Nancy's bloodstream if she drinks 9 grams of alcohol per hour.

$$a^* = a^* - \frac{9.5a^*}{4+a^*} + 9$$

$$\frac{9.5a^*}{4+a^*} = 9$$

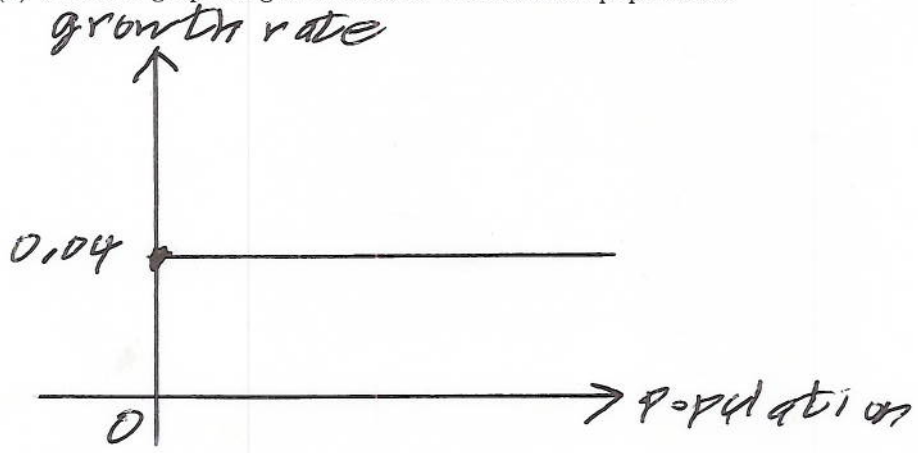
$$9.5a^* = 36 + 9a^*$$

$$0.5a^* = 36$$

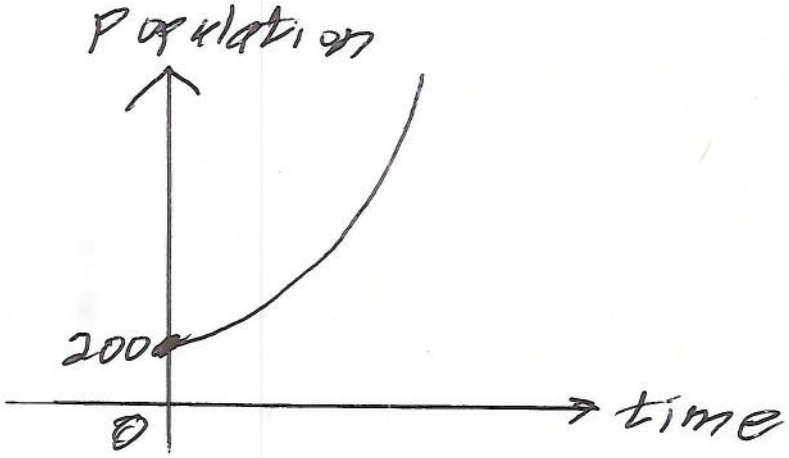
$$a^* = 72 \text{ grams}$$

8. (15 points) There are currently 200 deer, but the population is expected to grow exponentially by 4% each year from now on.

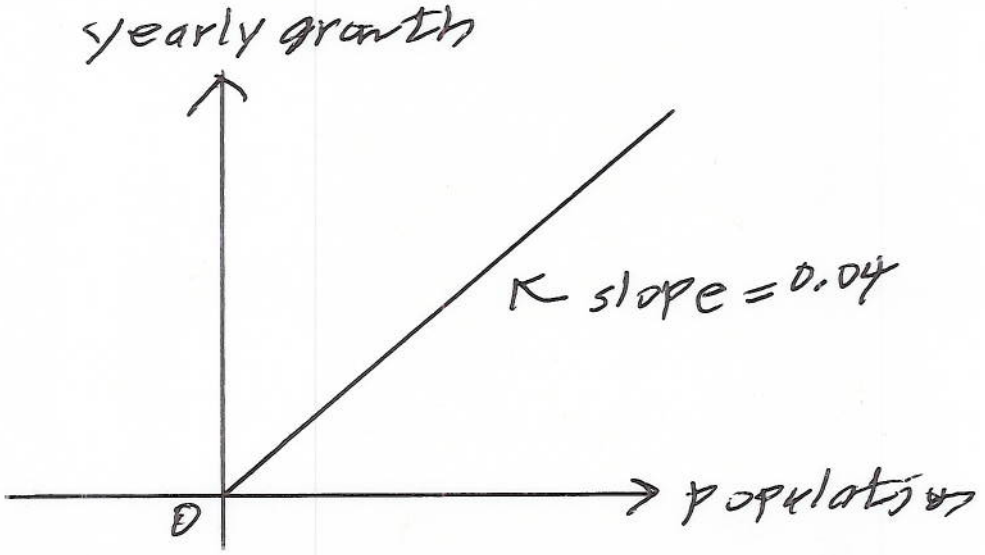
(a) Sketch a graph of growth rate as a function of population.



(b) Sketch a graph of population as a function of time.

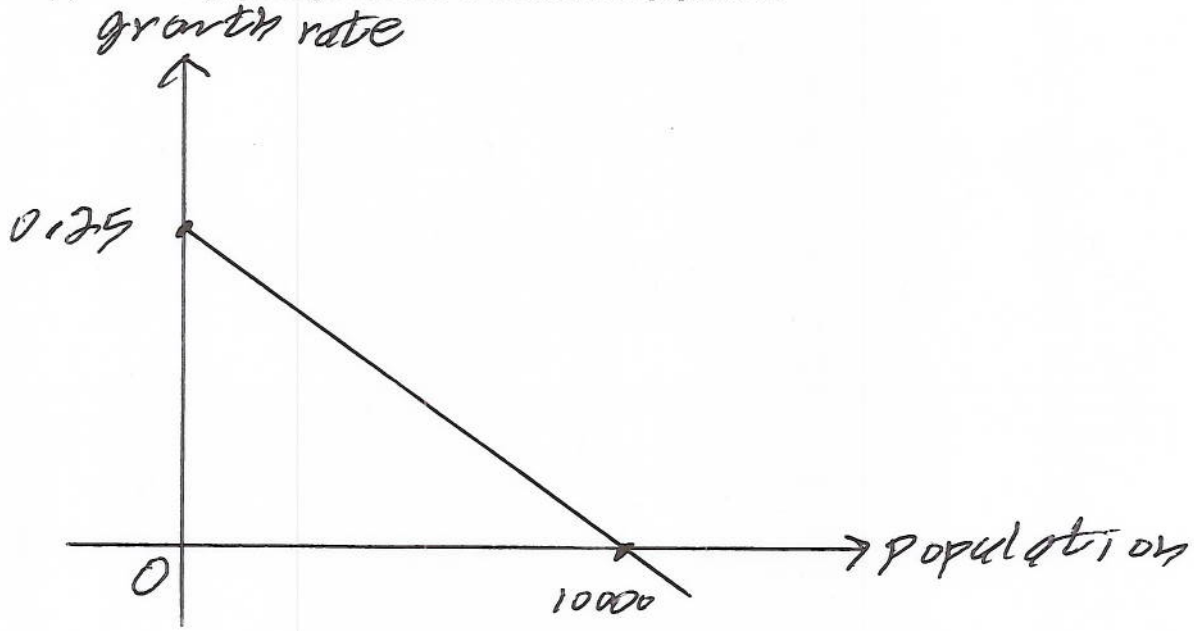


(c) Sketch a graph of yearly growth as a function of population.

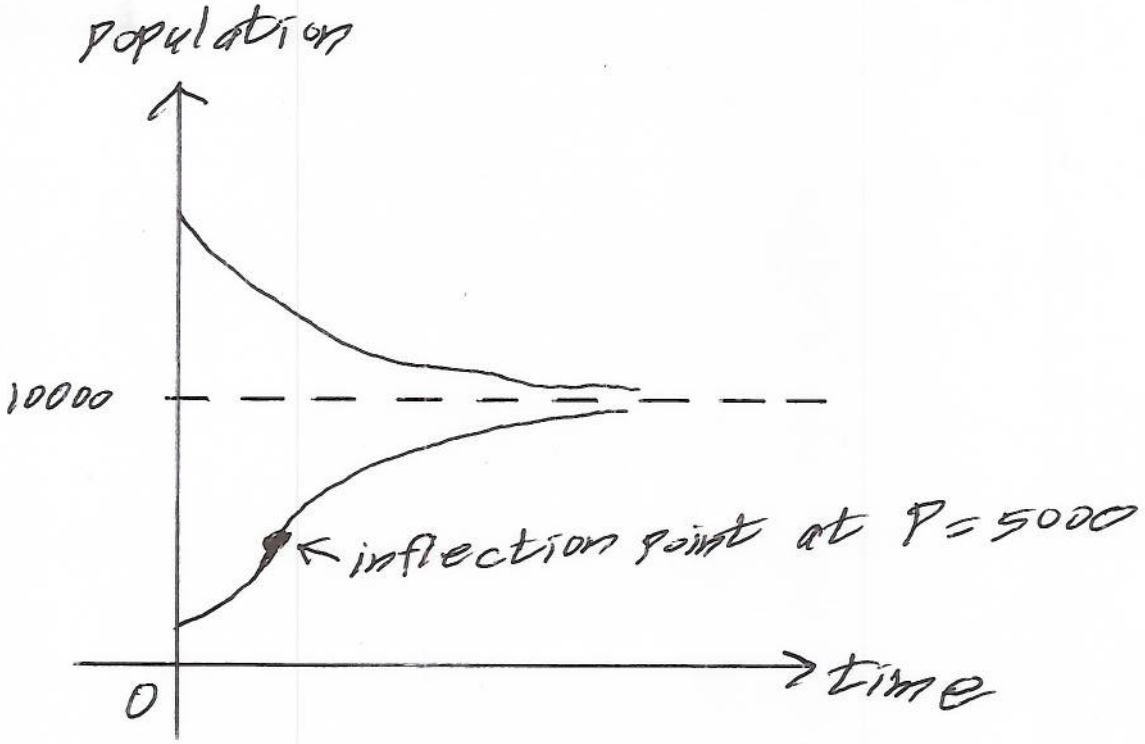


9. (12 points) Suppose that a bird population grows logistically with an intrinsic growth rate of 25% and a carrying capacity of 10,000.

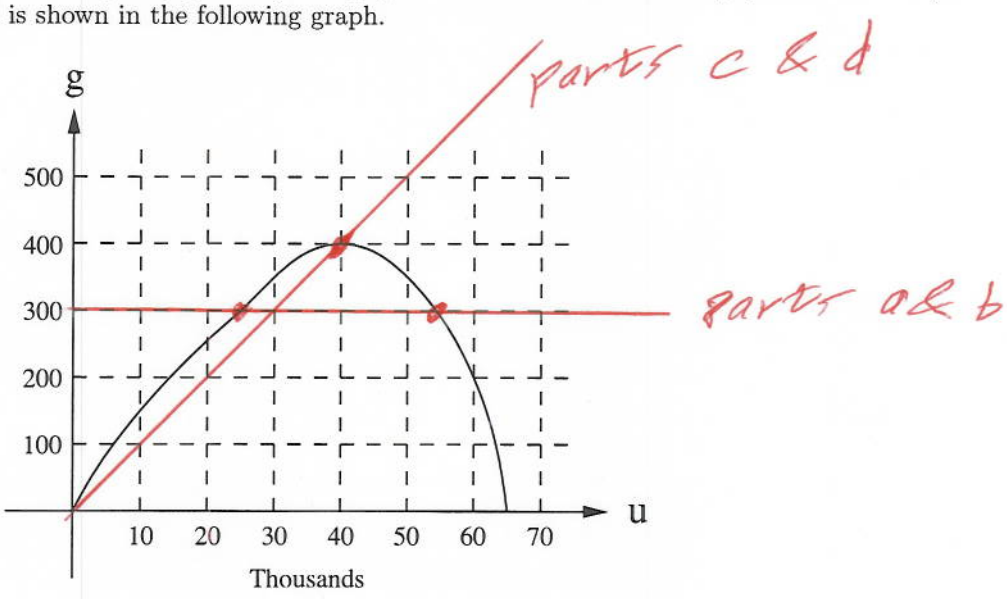
(a) Sketch a graph of growth rate as a function of population.



(b) Sketch a graph of population as a function of time.



10. (16 points) The natural yearly growth g in a population is a function of the population size u (in thousands) and is shown in the following graph.



(a) Estimate the stable equilibrium population if there is a constant yearly harvest of 300.

55000

(b) Estimate the minimum viable population if there is a constant yearly harvest of 300.

25000

(c) Estimate the maximum constant sustainable yearly harvest.

400

(d) Approximate the percent of the population that should be harvested each year to maximize the sustainable harvest.

$\frac{400}{40000} = 0.01$

1%