Math 172 Quiz 1 January 21, 2009

Name SOLUTIONS

1. (2 points) Given that $F(0) = 20$ and $F(n+1) = 0.84F(n) + 40$, make a table of values for F with $n = 0, 4, 8, 12,$ and 16. If you observe the behavior of this function for larger values of n, what long term behavior do you observe?

n	F
0	20
4	135.49
8	192.99
12	221.62
16	235.87
⋮	⋮
52	249.97
56	249.99
60	249.99
	↓
	250

F increases and levels off at 250

$$\lim_{n \to \infty} F(n) = 250$$

2. (2 points) Suppose that h is a function of t and takes on the values shown in the following table.

t	h
0	3
1	9
2	15
3	21
4	27
5	33

(a) Determine a discrete dynamical system along with an intial value for this function.

$$h(t) = h(t-1) + 6$$
$$h(0) = 3$$

(b) Determine an explicit formula for this function.

$$h(t) = 6t + 3$$

3. (3 points) Suppose that a patient takes a daily dose of 4.5 milligrams of some drug, and that each day the kidneys filter out 10% of this drug from the patient's bloodstream.

(a) Determine a discrete dynamical system along with an initial value for $d(t)$, the number of milligrams of this drug in this patient's bloodstream t days after the initial dose.

Let $d(t)$ be the number of milligrams of this drug in the bloodstream t days after the initial dose.

$$d(t) = d(t-1) - 0.1d(t-1) + 4.5$$

$$d(t) = 0.9d(t-1) + 4.5$$

$$d(0) = 4.5$$

(b) How many milligrams of this drug are in the bloodstream 5 days after the initial dose?

t	$d(t)$
0	4.5
1	8.55
2	12.195
3	15.476
4	18.428
5	21.085

$d(5) = 21.085$ mg

(c) Assuming that the patient is to continue taking this drug for a long time, what was the doctor's **target goal** for the desired amount of this drug in the bloodstream?

t	$d(t)$
0	4.5
20	40.076
40	44.401
60	44.927
80	44.991
100	44.999
	45

45 mg

since $\lim_{t\to\infty} d(t) = 45$

4. (3 points) There are currently 400 people living in a small town which is projected to grow by 8% each year. Let $P(t)$ represent the population of the town t years from now.

(a) Find a discrete dynamical system along with an initial value for $P(t)$.

$$P(t) = P(t-1) + 0.08P(t-1)$$

$$P(t) = 1.08P(t-1)$$

$$P(0) = 400$$

(b) Find an explicit formula for $P(t)$.

$$P(t) = 400(1.08)^t$$

(c) In what year will the population reach 2000 people?

$$2000 = 400(1.08)^t$$

$$5 = 1.08^t$$

$$\ln(5) = \ln(1.08^t)$$

$$\ln(5) = t \cdot \ln(1.08)$$

$$t = \ln(5)/\ln(1.08)$$

$t \approx$ 20.9 years from now

or use a table of values to approximate