1. Each dynamical system below has an equilibrium value of 50. Determine if this equilibrium value is stable or unstable for each system. You do not need to show your work here.

(a) $u(n) = 1.05u(n-1) - 2.5$

n	$u(n)$
0	15
5	5.33
10	-7.01
15	-22.76
20	-42.87
25	-68.52

n	$u(n)$
0	60
5	62.76
10	66.29
15	70.79
20	76.53
25	83.86

UNSTABLE

$\lim_{n\to\infty} u(n) \neq 50$

(b) $P(t) = 0.92P(t-1) + 4$

t	$P(t)$
0	10
10	32.62
20	42.45
30	46.72
40	48.58
50	49.38
60	49.73
70	49.88
80	49.95
	↓
	50

t	$P(t)$
0	75
10	60.86
20	54.72
30	52.05
40	50.89
50	50.39
60	50.17
70	50.07
80	50.03
	↓
	50

STABLE

$\lim_{t\to\infty} P(t) = 50$

(c) $v(n+1) = 0.96v(n) + 2$

n	$v(n)$
0	40
8	42.79
16	44.80
24	46.25
32	47.29
40	48.05
48	48.59
56	48.98
64	49.27
	↓
	50

n	$v(n)$
0	65
8	60.82
16	57.81
24	55.63
32	54.06
40	52.93
48	52.11
56	51.53
64	51.10
	↓
	50

STABLE

$\lim_{n \to \infty} v(n) = 50$

(d) $Q(t+1) = -0.9Q(t) + 95$

t	$Q(t)$
0	30
6	39.37
12	44.35
18	47.00
24	48.40
30	49.15
36	49.55
42	49.76
	↓
	50

t	$Q(t)$
0	55
6	52.66
12	51.41
18	50.75
24	50.40
30	50.21
36	50.11
42	50.06
	↓
	50

STABLE

$\lim_{t \to \infty} Q(t) = 50$

2. Find the equilibrium value for the following dynamical system. Show your work.

$$P(t+1) = 0.93P(t) + 17.5$$

$P^* = 0.93P^* + 17.5$

$0.07P^* = 17.5$

$P^* = \frac{17.5}{0.07}$

$P^* = 250$

CHECK ON CALCULATOR

t	$P(t)$
0	250
1	250
2	250
⋮	⋮

✓

3. Find the equilibrium point for the following dynamical system. Show your work.

$$u(n) = 0.8u(n-1) + 0.6v(n-1) - 3.2$$
$$v(n) = 0.8u(n-1) + 0.9v(n-1) - 14.8$$

$u^* = 0.8u^* + 0.6v^* - 3.2$

$v^* = 0.8u^* + 0.9v^* - 14.8$

$10u^* = 8u^* + 6v^* - 32$

$10v^* = 8u^* + 9v^* - 148$

$2u^* = 6v^* - 32$

$v^* = 8u^* - 148$

$2u^* = 6(8u^* - 148) - 32$

$2u^* = 48u^* - 888 - 32$

$920 = 46u^*$

$u^* = \frac{920}{46} = 20$

$v^* = 8(20) - 148 = 12$

equilibrium value is $(u^*, v^*) = (20, 12)$

CHECK ON CALCULATOR

n	$u(n)$	$v(n)$
0	20	12
1	20	12
2	20	12
⋮	⋮	⋮

✓

4. A doctor prescribes some drug to be taken every 6 hours. Suppose the body eliminates 12.5% of this drug every 6 hours. What should the prescribed dosage of this drug be if the doctor wants the equilibrium value (the target goal) to be equal to 160 ml ?

LET $d(n)$ BE THE NUMBER OF ml OF THIS DRUG IN THE BLOODSTREAM IN 6-HOUR PERIODS AFTER THE INITIAL DOSE.

$$d(n) = d(n-1) - 0.125\,d(n-1) + x$$

$$d(n) = 0.875\,d(n-1) + x$$

$$d^* = 0.875\,d^* + x$$

$$160 = 0.875(160) + x$$

$$160 = 140 + x$$

$$x = 20 \text{ ml}$$

5. Suppose you borrow \$80,000 at an 6% annual interest rate compounded monthly to be paid back in monthly payments of \$1600.

(a) Write down a discrete dynamical system with initial condition to represent the balance of the loan just after each month's payment.

$b(n) = b(n-1) + \text{INTEREST} - \text{PAYMENT}$

$b(n) = b(n-1) + \frac{0.06}{12} b(n-1) - 1600$

$b(n) = 1.005\, b(n-1) - 1600$

$b(0) = 80000$

(b) How many months will it take to pay back the loan?

n	$b(n)$
0	80000
1	78800
⋮	
57	1083.67
58	−510.91

58 months

(c) The last payment will be a bit different than each of the preceding monthly payments. To the nearest penny, what will be the amount of this last payment?

FIRST 57 PAYMENTS ARE EACH \$1600.

THE 58th AND FINAL PAYMENT IS

$1600 - 510.91 = \$1089.09$