Math 172 **Quiz 3** **February 9, 2009**

Name SOLUTIONS

1. (4 points) The dynamical system shown has a stable equilibrium point at $(p^*, q^*) = (100, 75)$. Given that $p(0) = 15$ and $q(0) = 20$, determine the eventual rate at which p approaches equilibrium. Show all calculations you made to find the rate.

$$p(t) = 0.7p(t-1) + 0.2q(t-1) + 15$$
$$q(t) = 0.1p(t-1) + 0.6q(t-1) + 20$$

$$\left|\frac{P(1)-100}{P(0)-100}\right| \approx 0.8294117647$$

$$\left|\frac{P(2)-100}{P(1)-100}\right| \approx 0.8177304965$$

$$\left|\frac{P(3)-100}{P(2)-100}\right| \approx 0.8108412836$$

$$\vdots$$

$$\left|\frac{P(11)-100}{P(10)-100}\right| \approx 0.8002438126$$

$$\vdots$$

$$\left|\frac{P(21)-100}{P(20)-100}\right| \approx 0.8000022157$$

$$\vdots$$

$$\left|\frac{P(31)-100}{P(30)-100}\right| \approx 0.8000000201$$

$$\lim_{t\to\infty}\left|\frac{P(t)-100}{P(t-1)-100}\right| = 0.8$$

EVENTUALLY, P GETS 20% CLOSER TO ITS EQUILIBRIUM EACH TIME PERIOD

2. (6 points) For the following discrete dynamical systems,

- Find an explicit formula for the given function.
- Determine the equilibrium value, or state that no equilibrium value exists.
- For systems with an equilibrium value, state whether the equilibrium value is stable or unstable.
- For systems with an equilibrium value, determine the rate at which the function approaches or moves away from equilibrium.

(a) $q(t+1) = q(t) - 3$ and $q(0) = 60$

- $q(t) = -3t + 60$
- NO EQUILIBRIUM VALUE SINCE LINEAR WITH NONZERO SLOPE
- X
- X

(b) $u(n) = 1.1u(n-1)$ and $u(0) = 40$

- $u(n) = 40(1.1)^n$
- $u^* = 0$ AS AN UNSTABLE ~~STABLE~~ EQUILIBRIUM VALUE SINCE $|1.1| > 1$
-
- $$\lim_{n \to \infty} \frac{u(n) - 0}{u(n-1) - 0} = \lim_{n \to \infty} \frac{1.1u(n-1)}{u(n-1)} = \lim_{n \to \infty} 1.1 = 1.1$$

SO u MOVES 10% FURTHER AWAY FROM EQUILIBRIUM EACH TIME PERIOD.

(c) $P(t) = 0.92P(t-1) + 16$ and $P(0) = 25$

$P^* = 0.92P^* + 16$

$0.08P^* = 16$

$P^* = \frac{16}{0.08} = 200$

$P(t) = c(0.92)^t + 200$

$25 = c(0.92)^0 + 200$

$25 = c + 200$

$c = -175$

- $P(t) = -175(0.92)^t + 200$
- $P^* = 200$
- $P^* = 200$ IS A STABLE EQUIL. VALUE SINCE $-1 < 0.92 < 1$
- $\lim_{t\to\infty} \left| \frac{P(t) - 200}{P(t-1) - 200} \right| = \lim_{t\to\infty} \left| \frac{0.92P(t-1) + 16 - 200}{P(t-1) - 200} \right|$

$= \lim_{t\to\infty} \left| \frac{0.92P(t-1) - 184}{P(t-1) - 200} \right|$

$= \lim_{t\to\infty} \left| \frac{0.92(P(t-1) - 200)}{P(t-1) - 200} \right|$

$= 0.92$

P MOVES 8% CLOSER TO ITS EQUILIBRIUM EACH TIME PERIOD