Math 172 **Quiz 4** **Spring 2009**

Name SOLUTIONS

1. (5 points) It has been raining heavily since midnight. Let $f(t)$ represent the total number of inches of rain which have fallen in the t hours since midnight. Given that $f'(2) = 4$, which one of the following sentences must be true?

   (a) From midnight to 2:00AM it rained a total of 4 inches.

   (b) From midnight to 4:00AM it rained a total of 2 inches.

   (c) From midnight to 2:00AM it was raining at an average rate of 4 inches per hour.

   (d) From midnight to 4:00AM it was raining at an average rate of 2 inches per hour.

   (e) At 2:00AM it was raining at a rate of 4 inches per hour.

   (f) At 4:00AM it was raining at a rate of 2 inches per hour.

2. A model for the population of a town predicts the population t years from now to be given by $P(t) = 850e^{-0.04t}$.

   (a) (1 point) What population does this model predict for this town 15 years from now?

   $P(15) = 850e^{-0.04(15)}$

   ≈ 466.4898907

   $\approx$ 466 people

   (b) (2 points) How quickly in people per year is the population predicted to be changing 15 years from now?

   $P'(t) = 850e^{-0.04t} \cdot (-0.04)$

   $P'(t) = -34e^{-0.04t}$

   $P'(15) = -34e^{-0.04(15)}$

   ≈ -18.7 people/year

3. (2 points) Find the equation of the line tangent to the graph of $y = x^3 - 10x$ at $x = 2$.

$y = x^3 - 10x \Rightarrow y(2) = -12$

$y' = 3x^2 - 10 \Rightarrow y'(2) = 2$

POINT: $(2, -12)$

SLOPE: 2

$y - (-12) = 2(x - 2)$

$y = 2x - 16$

4. (3 points) Find derivatives of the following functions. Use Leibniz notation for the derivative.

(a) $y = \ln\left(2x^5 + 8x^2 + 15\right)$

$$\frac{dy}{dx} = \frac{1}{2x^5 + 8x^2 + 15} \cdot (10x^4 + 16x)$$

$$\frac{dy}{dx} = \frac{10x^4 + 16x}{2x^5 + 8x^2 + 15}$$

(b) $h = \frac{9}{r^3} = 9r^{-3}$

$$\frac{dh}{dr} = -27r^{-4}$$

(c) $P = 4t^3e^{-t}$

$$\frac{dP}{dt} = (4t^3)'(e^{-t}) + (4t^3)(e^{-t})'$$

$$\frac{dP}{dt} = 12t^2e^{-t} + 4t^3(e^{-t} \cdot (-1))$$

$$\frac{dP}{dt} = 12t^2e^{-t} - 4t^3e^{-t}$$