Math 172 **Quiz 6 Solutions** **Spring 2009**

1. Since $\frac{dp}{dr} = 2(r-3)^2(r-5)$ is positive for $r > 5$ and is zero for $r = 5$, we see that p is increasing for $r \in [5, \infty)$.

2. Since $\frac{dw}{dt} = 3(w^3 - 4w)(w-6) = 3w(w-2)(w+2)(w-6)$ is negative for $-2 < w < 0$ or $2 < w < 6$, and is zero at the endpoints of these intervals, we see that w is decreasing for $w \in [-2, 0] \cup [2, 6]$.

3. Solving $0 = 0.5P(P^2 - 9)(P^2 + 4)(P^2 - 2) = 0.5P(P-3)(P+3)(P^2+4)(P - \sqrt{2})(P + \sqrt{2})$ gives equilibrium values at $P = 0$, $P = 3$, $P = -3$, $P = \sqrt{2}$, $P = -\sqrt{2}$.

4. I will describe the graphs since I don't currently have access to the department's PDF scanner.

   (a) With initial value $y(0) = 10$, the graph starts at the point $(0, 10)$, decreases, and approaches the horizontal asymptote at $y = 8$.

   (b) With initial value $y(0) = 8$, the graph is the horizontal line at $y = 8$.

   (c) With initial value $y(0) = 6$, the graph starts at the point $(0, 6)$, increases, and approaches the horizontal asymptote at $y = 8$.

   (d) With initial value $y(0) = 4$, the graph is the horizontal line at $y = 4$.

   (e) With initial value $y(0) = 2$, the graph starts at the point $(0, 2)$, increases, and approaches the horizontal asymptote at $y = 4$.

5. Let P represent the population of this city t years after 1990.

   (a) $\frac{dP}{dt} = 80 \quad$ and $\quad P(0) = 4000$

   (b) $P(t) = P(t-1) + 80 \quad$ and $\quad P(0) = 4000$

   (c) $P = 80t + 4000$

   (d) $P(15) = 5200$ people

   (e) Solving $12000 = 80t + 4000$ we get $t = 100$ so this population is reached in the year 2090.

6. Let A represent the amount in this account t years after the initial deposit.

   (a) Since interest is compounded continuously we will use the following differential equation to model the amount of money in this account.
   $\frac{dA}{dt} = 0.06A \quad$ and $\quad A(0) = 500$

   (b) $A = 500e^{0.06t}$

   (c) $A(9) \approx \$858.00$

   (d) Solving $1500 = 500e^{0.06t}$ we obtain $t = \frac{\ln 3}{0.06} \approx 18.3$ years

7. Let A represent the amount in this account t years after the initial deposit.

   (a) Since interest is compounded annually we will use the following discrete dynamical system to model the amount of money in this account.
   $A(t) = A(t-1) + 0.08A(t-1) = 1.08A(t-1)$ and $A(0) = 600$

   (b) $A = 600(1.08)^t$

   (c) $A(6) \approx \$952.12$

   (d) Solving $1500 = 600(1.08)^t$ we obtain $t = \dfrac{\ln 2.5}{\ln 1.08} \approx 11.9$ years

8. Let D represent the population of deer t years from now.

   (a) I will describe the graph since I don't currently have access to the department's PDF scanner. The graph starts at the point $(0, 6000)$, increases, and approaches the horizontal asymptote at $D = 20000$. Furthermore there is an inflection point when the population is $D = 10000$. The graph is concave up before this point and concave down afterwards.

   (b) $D = 10000$ deer which gives $\dfrac{dD}{dt} = 250$ deer per year

   (c) $D(t) = D(t-1) + 0.05D(t-1)\left(1 - \dfrac{D(t-1)}{20000}\right)$ and $D(0) = 6000$

   (d) $\dfrac{dD}{dt} = 0.05D\left(1 - \dfrac{D}{20000}\right)$ and $D(0) = 6000$

   (e)

$t_{current}$	$D_{current}$	$D'_{current}$	$D_{next} \approx D_{current} + D'_{current} \cdot \Delta t$
0	6000.00	210.00	6210.00
1	6210.00	214.09	6424.09
2	6424.09	218.03	6642.12
3	6642.12	221.81	6863.93
4	6863.93		

   Note that since $\Delta t = 1$, our table has the same values that would be generated by the discrete dynamical system.

   (f)

$t_{current}$	$D_{current}$	$D'_{current}$	$D_{next} \approx D_{current} + D'_{current} \cdot \Delta t$
0.0	6000.00	210.00	6105.00
0.5	6105.00	212.07	6211.04
1.0	6211.04	214.11	6318.09
1.5	6318.09	216.11	6426.15
2.0	6426.15	218.07	6535.18
2.5	6535.18	219.99	6645.17
3.0	6645.17	221.86	6756.10
3.5	6756.10	223.69	6867.95
4.0	6867.95		