

Name _____

1. Suppose that p is a function of r which satisfies the differential equation

$$\frac{dp}{dr} = 2(r - 3)^2(r - 5)$$

Upon which one of the following intervals must p be increasing?

- (a) $r \in (-\infty, 3]$
 - (b) $p \in (-\infty, 3]$
 - (c) $r \in [-3, 3]$
 - (d) $p \in [-3, 3]$
 - (e) $r \in [3, 5]$
 - (f) $p \in [3, 5]$
 - (g) $r \in [5, \infty)$
 - (h) $p \in [5, \infty)$
2. Suppose that w is a function of t which satisfies the differential equation

$$\frac{dw}{dt} = 3(w^3 - 4w)(w - 6)$$

Upon which one of the following intervals must w be decreasing?

- (a) $t \in (-\infty, 2]$
 - (b) $w \in (-\infty, 2]$
 - (c) $t \in (-\infty, 6]$
 - (d) $w \in (-\infty, 6]$
 - (e) $t \in [-2, 0] \cup [2, 6]$
 - (f) $w \in [-2, 0] \cup [2, 6]$
 - (g) $t \in [0, 2]$
 - (h) $w \in [0, 2]$
 - (i) $t \in [2, \infty)$
 - (j) $w \in [2, \infty)$
3. Find all equilibrium values for the following differential equation. There is no need to discuss whether or not these equilibrium values are stable.

$$\frac{dP}{dt} = 0.5P(P^2 - 9)(P^2 + 4)(P^2 - 2)$$

4. Suppose y is a function of t which satisfies the differential equation below.

$$\frac{dy}{dt} = 0.25y(y - 4)^2(8 - y)$$

Sketch plausible graphs for y as a function of t given each initial value below. Your graphs should clearly show if the y -values approach any particular values (i.e. horizontal asymptotes). You should draw all five graphs together on one set of coordinate axes.

(a) $y(0) = 10$

(b) $y(0) = 8$

(c) $y(0) = 6$

(d) $y(0) = 4$

(e) $y(0) = 2$

5. The population of a city was 4000 in 1990. Since then the population has been increasing by 80 people per year.

(a) Determine a differential equation with initial value to model the city's population.

(b) Determine a discrete dynamical system with initial value to model the city's population.

(c) Determine an explicit formula for the city's population.

(d) What does your model predict for the city's population in the year 2005?

(e) When does your model predict the population will have reached 12000?

6. An initial deposit of \$500 is made into an account with an annual interest rate of 6% compounded continuously.
- (a) Is it more appropriate to use a discrete dynamical system or a differential equation to model the amount of money in this account? Write the equations to describe your model.
 - (b) Determine an explicit formula for the amount of money in this account.
 - (c) How much money will the account hold 9 years after the initial deposit?
 - (d) How long will it take until the balance in this account is \$1500?
7. An initial deposit of \$600 is made into an account with an annual interest rate of 8% compounded annually.
- (a) Is it more appropriate to use a discrete dynamical system or a differential equation to model the amount of money in this account? Write the equations to describe your model.
 - (b) Determine an explicit formula for the amount of money in this account.
 - (c) How much money will the account hold 6 years after the initial deposit?
 - (d) How long will it take until the balance in this account is \$1500?

8. There are currently 6000 deer in a forest. Suppose the population of deer grows logistically with an intrinsic growth rate of 5% and a carrying capacity of 20,000.

(a) Sketch a rough graph of the deer population.

(b) Find the population and the rate of change of population when the population is increasing most rapidly (in deer per year).

(c) Determine a discrete dynamical system with initial value to model the deer population.

(d) Determine a differential equation with initial value to model the deer population.

(e) Use Euler's Method with $\Delta t = 1$ to make a table of values for the number of deer your continuous model predicts for the next 4 years.

(f) Use Euler's Method with $\Delta t = 0.5$ to make a table of values for the number of deer your continuous model predicts for the next 4 years.