Math 172 **Worksheet D** **Spring 2009**

1. For each discrete dynamical system do the following.

   - Quickly decide if the system represents a linear, exponential, or shifted exponential function.
   - Quickly decide if the system has a stable equilibrium value, an unstable equilibrium value, or no equilibrium value.
   - Find an explicit formula for the system.
   - For each system that has an equilibrium value, determine as a percentage how much closer to or further away from the equilibrium value the function moves each time period.

   (a) $P(t+1) = P(t) - 20$ and $P(0) = 350$

   (b) $v(n+1) = \dfrac{v(n)}{4}$ and $v(0) = 1600$

   (c) $q(t) = q(t-1) + 2.5$ and $q(0) = 40$

   (d) $h(s) = \dfrac{3}{2}h(s-1)$ and $h(0) = 45$

   (e) $Q(t+1) = 1.25Q(t) - 22.5$ and $Q(0) = 150$

   (f) $B(n) = 0.9B(n-1) + 30$ and $B(0) = 750$

   (g) $d(n) = 1.3d(n-1) - 63$ and $d(0) = 250$

   (h) $w(t) = 0.85w(t-1) + 90$ and $w(0) = 200$

   (i) $p(n) = \dfrac{p(n-1) - 2500}{5}$ and $p(0) = 50$

2. The dynamical system shown has a stable equilibrium point at $(p^*, q^*) = (280, 300)$. Given that $p(0) = 50$ and $q(0) = 23$, determine the eventual rate at which q approaches equilibrium. Show all calculations you made to find the rate.

$$p(t) = 0.5p(t-1) + 0.3q(t-1) + 50$$
$$q(t) = 0.4p(t-1) + 0.55q(t-1) + 23$$

3. The dynamical system shown has a stable equilibrium point at $(u^*, v^*) = (20, 32)$. Given that $u(0) = 10$ and $v(0) = 20$, determine the eventual rate at which u approaches equilibrium and the rate at which v approaches equilibrium. Show all calculations you made to find the rate.

$$u(n) = 0.3u(n-1) - 0.5v(n-1) + 30$$
$$v(n) = 0.2u(n-1) + v(n-1) - 4$$

4. For the following dynamical system, there is no equilibrium point, but the values for $u(n)$ eventually change by approximately the same amount.

$$u(n) = 0.9u(n-1) + 0.2v(n-1) + 600$$
$$v(n) = 0.1u(n-1) + 0.8v(n-1) + 400$$

   (a) What is that approximate amount by which $u(n)$ eventually changes?

   (b) Does $u(n)$ appear to be more linear or exponential for large n?

5. For the following dynamical system, there is no equilibrium point, but the values for $p(t)$ eventually change by approximately the same amount.

$$p(t) = 1.2p(t-1) + 0.6q(t-1) + 10$$
$$q(t) = -0.2p(t-1) + 0.4q(t-1) + 20$$

   (a) What is that approximate amount by which $p(t)$ eventually changes?

   (b) Does $p(t)$ appear to be more linear or exponential for large t?

   (c) Answer questions (a) and (b) for the function $q(t)$.