1. Write down an equation to show that the circumference of a circle is proportional to its radius. What is the constant of proportionality?

2. Write down an equation to show that the area of an equilateral triangle is proportional to the square of its side length. What is the constant of proportionality?

3. After the brakes are applied in an automobile, it will still travel a certain distance before coming to rest. This is referred to as the automobile's *stopping distance*, and it is directly proportional to the square of the automobile's speed. If an automobile has a stopping distance of 45 feet when traveling at 30 miles per hour, then what is the stopping distance of the same automobile traveling at 60 miles per hour?

4. The population of a town is currently 4000. Letting P represent the town's population t years from now, write down a differential equation with initial value to model the population under the following conditions.

   (a) The population is growing at a rate of 40 people per year.

   (b) The population is growing at a rate which is proportional to the population size with a constant of proportionality of 0.05.

5. Suppose that 500 northern pike are released into a man-made lake which had no northern pike beforehand. Write down a differential equation with initial value to model the number of these fish under the following conditions.

   (a) This fish population decreases by 40 fish per year.

   (b) This fish population grows at a continuous growth rate of 6% per year.

6. Alice was standing in a room with a 12 foot ceiling. She is normally only 4 feet tall, but after drinking liquid from a strange bottle, she started to grow at a rate which is proportional to the product of her height and the distance from the top of her head to the ceiling. If h represents Alice's height at time t, then find the differential equation which models her height.

7. Newton's Law of Cooling states that the rate at which an object cools is proportional to the difference between its temperature and that of its surroundings.

   (a) Using T for temperature at time t, k for the constant of proportionality, and T_s for the surrounding temperature, determine a differential equation which models the object's temperature.

   (b) A fresh cup of coffee has a temperature of $90°C$ and is brought into a room where the temperature is $20°C$. Suppose k has the value of $-0.1°C$ per minute per $°C$ of temperature difference. Write down the differential equation which models the coffee's temperature.

8. Suppose y is a function of x which satisfies the following differential equation.

$$\frac{dy}{dx} = 2x, \qquad y(0) = 1$$

(a) Use Euler's Method with $\Delta x = 1$ to approximate $y(2)$.

$x_{current}$	$y_{current}$	$y'_{current}$	$y_{next} \approx y_{current} + y'_{current} \cdot \Delta x$
0.0	1		
1.0			
2.0			

(b) Use Euler's Method with $\Delta x = 0.5$ to approximate $y(2)$.

$x_{current}$	$y_{current}$	$y'_{current}$	$y_{next} \approx y_{current} + y'_{current} \cdot \Delta x$
0.0	1		
0.5			
1.0			
1.5			
2.0			

(c) Use Euler's Method with $\Delta x = 0.1$ to approximate $y(2)$.

$x_{current}$	$y_{current}$	$y'_{current}$	$y_{next} \approx y_{current} + y'_{current} \cdot \Delta x$
0.0	1		
0.1			
0.2			
0.3			
0.4			
0.5			
0.6			
0.7			
0.8			
0.9			
1.0			
1.1			
1.2			
1.3			
1.4			
1.5			
1.6			
1.7			
1.8			
1.9			
2.0			

(d) Find an explicit formula for y which satisfies the differential equation. Use the formula to find the exact value of $y(2)$. Compare your approximations in parts (a) – (d).

9. On problem **7b** we found the coffee's temperature to be modeled by the following differential equation.

$$\frac{dT}{dt} = -0.1(T - 20), \qquad T(0) = 90$$

(a) Use Euler's method with the value of Δt shown to approximate the coffee's temperature after 10 minutes.

$t_{current}$	$T_{current}$		
0.0	90	-7	72.5
2.5	72.5		
5.0			
7.5			
10.0			

(b) Be sure to make a couple of additional tables with smaller values chosen for Δt. If you are proficient at computer programming or using spreadsheets such as Excel, then your smallest value for Δt may be as small as 0.01. The rest of us should at least be willing to use $\Delta t = 0.5$.

10. Suppose P is a function of t which satisfies the following differential equation.

$$\frac{dP}{dt} = 0.1P, \qquad P(0) = 100$$

(a) Make tables similar to those used in problem **8** to approximate $P(3)$ using Euler's Method.

(b) Can you find an explicit formula for P which satisfies the differential equation? If so, then use the formula to find the exact value of $P(3)$. How do your approximations compare to exact value?

11. Use Euler's method to estimate the population 6 years from now in problems **4a**, **4b**, **5a**, and **5b**. Use $\Delta t = 2$, $\Delta t = 1$, and $\Delta t = 0.5$ for each problem. Find an explicit formula for the population in each of these problems. How do your estimates compare to the exact populations 6 years from now?