

1. Suppose that y is a function of x which satisfies the differential equation

$$\frac{dy}{dx} = 3(x - 2)(x - 4)$$

- (a) Specify an interval of either x -values or y -values for which the graph of y will be increasing.
(b) Specify an interval of either x -values or y -values for which the graph of y will be decreasing.

2. Suppose that y is a function of x which satisfies the differential equation

$$\frac{dy}{dx} = 3(y - 2)(y - 4)$$

- (a) Specify an interval of either x -values or y -values for which the graph of y will be increasing.
(b) Specify an interval of either x -values or y -values for which the graph of y will be decreasing.

3. Consider the population model for logistic growth

$$\frac{dP}{dt} = 0.15P \left(1 - \frac{P}{6000} \right)$$

where P is the population at time t .

- (a) For which values of P is the population increasing?
(b) For which values of P is the population decreasing?
(c) For which values of P is the population in equilibrium? Determine whether each of these equilibrium values is stable or unstable.
(d) Sketch a rough graph of P given that: (i) $P(0) = 500$, (ii) $P(0) = 4000$, (iii) $P(0) = 6000$, (iv) $P(0) = 8000$.

4. Consider the population model

$$\frac{dP}{dt} = 0.3P \left(1 - \frac{P}{200} \right) \left(\frac{P}{50} - 1 \right)$$

where P is the population at time t .

- (a) For which values of P is the population increasing?
(b) For which values of P is the population decreasing?
(c) For which values of P is the population in equilibrium? Determine whether each of these equilibrium values is stable or unstable.
(d) Sketch a rough graph of P given that: (i) $P(0) = 10$, (ii) $P(0) = 50$, (iii) $P(0) = 150$, (iv) $P(0) = 300$.

5. Find all equilibrium values for the following differential equation. There is no need to discuss whether or not these equilibrium values are stable.

$$\frac{dP}{dt} = 0.5(P - 4)(P + 3)(P^2 - 25)$$

6. Suppose y is a function of t which satisfies the differential equation below.

$$\frac{dy}{dt} = 0.25(y - 10)(20 - y)$$

Sketch plausible graphs for y as a function of t given each initial value below. Your graphs should clearly show if the y -values approach any particular values (i.e. horizontal asymptotes). You should draw all five graphs together on one set of coordinate axes.

(a) $y(0) = 25$

(b) $y(0) = 20$

(c) $y(0) = 15$

(d) $y(0) = 10$

(e) $y(0) = 5$

3 Important Growth Models

- LINEAR GROWTH ($m = \text{slope}$, $P_0 = \text{initial value}$)

Type	Linear Model	Explicit Solution
continuous	$\frac{dP}{dt} = m$	$P = mt + P_0$
discrete	$P(t) = P(t - 1) + m$	$P = mt + P_0$

- EXPONENTIAL GROWTH ($r = \text{growth rate}$, $P_0 = \text{initial value}$)

Type	Exponential Model	Explicit Solution
continuous	$\frac{dP}{dt} = rP$	$P = P_0 e^{rt}$
discrete	$P(t) = P(t - 1) + rP(t - 1)$	$P = P_0 (1 + r)^n$

- LOGISTIC GROWTH ($r = \text{growth rate}$, $k = \text{carrying capacity}$, $P_0 = \text{initial value}$)

Type	Logistic Model
continuous	$\frac{dP}{dt} = rP \left(1 - \frac{P}{k}\right)$
discrete	$P(t) = P(t - 1) + rP(t - 1) \left(1 - \frac{P(t - 1)}{k}\right)$

7. If P represents some population t years from now, and $\frac{dP}{dt} = 20$, then which of the following statements is correct?
- (a) P grows linearly by 20 people per year.
 - (b) P grows linearly by 120 people per year.
 - (c) P grows exponentially by 20% per year.
 - (d) P grows logistically with a carrying capacity of 20.
8. If P represents some population t years from now, and $P(t) = 1.2P(t - 1)$, then which of the following statements is correct?
- (a) P grows linearly by 20 people per year.
 - (b) P grows linearly by 120 people per year.
 - (c) P grows exponentially by 20% per year.
 - (d) P grows logistically with a carrying capacity of 120.
9. The population of a city was 5000 in 1980. Since then the population has been increasing by 100 people per year.
- (a) Determine a discrete dynamical system with initial value to model the city's population.
 - (b) Determine a differential equation with initial value to model the city's population.
 - (c) Determine an explicit formula for the city's population.
 - (d) What does your model predict for the city's population in the year 2000?
 - (e) When does your model predict the population will have reached 10000?
10. An initial deposit of \$200 is made into an account with an annual interest rate of 3% compounded annually.
- (a) Determine a discrete dynamical system with initial value to model the amount of money in this account.
 - (b) Determine an explicit formula for the amount of money in this account.
 - (c) How much money will the account hold 8 years after the initial deposit?
 - (d) How long will it take until the balance in this account is \$500?
11. An initial deposit of 200 is made into an account with an annual interest rate of 3% compounded continuously.
- (a) Determine a differential equation with initial value to model the amount of money in this account.
 - (b) Determine an explicit formula for the amount of money in this account.
 - (c) How much money will the account hold 8 years after the initial deposit?
 - (d) How long will it take until the balance in this account is \$500?

12. Why did I suggest using a discrete dynamical system for problem (10), but a differential equation for problem (11)?
13. The number of fish in a pond was 300 in 1970. Since then the number of fish has been increasing by 5% per year.
 - (a) Determine a discrete dynamical system with initial value to model the number of fish in this pond.
 - (b) Enter this system into your calculator to make a table of values for the number of fish in the pond each year from 1970 to 1980.
 - (c) Determine an explicit formula for the number of fish in the pond.
 - (d) When will the fish population reach 750?
14. The number of fish in a pond was 300 in 1970. Since then the number of fish has been increasing by 5% per year.
 - (a) Determine a differential equation with initial value to model the number of fish in this pond.
 - (b) Use Euler's Method with $\Delta t = 1$ to make a table of values for the number of fish in the pond each year from 1970 to 1980.
 - (c) Determine an explicit formula for the number of fish in the pond.
 - (d) When will the fish population reach 750?
15. Which model was best to use for the fish population – the discrete dynamical system or the differential equation? Why?
16. There are currently 5000 deer in a forest. Suppose the population of deer grows logistically with an intrinsic growth rate of 6% and a carrying capacity of 20,000.
 - (a) Sketch a rough graph of the deer population.
 - (b) Determine a discrete dynamical system with initial value to model the deer population.
 - (c) Make a table of values for the number of deer your discrete model predicts for the next 4 years.
 - (d) Determine a differential equation with initial value to model the deer population.
 - (e) Use Euler's Method with $\Delta t = 1$ to make a table of values for the number of deer your continuous model predicts for the next 4 years.
 - (f) Use Euler's Method with $\Delta t = 0.5$ to make a table of values for the number of deer your continuous model predicts for the next 4 years.
 - (g) How close are the approximations for discrete and continuous models?