1. In the discrete dynamical system below, find each equilibrium value and determine if it is stable or unstable.

$$u(n) = 0.3u^2(n-1) - 2.6u(n-1) + 9.6$$

2. In the discrete dynamical system below, find each equilibrium value and determine if it is stable or unstable.

$$u(n) = 0.01u^2(n-1) + 0.3u(n-1) + 10$$

3. In the discrete dynamical system below, find each equilibrium value and determine if it is stable or unstable. For each stable equilibrium value, determine the maximum interval of stability.

$$u(n) = 1.2u(n-1) - 0.0004u^2(n-1)$$

4. In the discrete dynamical system below, find each equilibrium value and determine if it is stable or unstable. For each stable equilibrium value, determine the maximum interval of stability.

$$u(n) = 1.4u(n-1) - 0.004u^2(n-1)$$

5. In the discrete dynamical system below, find each equilibrium value and determine if it is stable or unstable. For each stable equilibrium value, determine the maximum interval of stability.

$$u(n) = 0.1u^2(n-1) + 0.3u(n-1) + 1$$

6. Suppose that a population of deer grows logistically with an intrinsic growth rate of 25% and a carrying capacity of 1600.

   (a) Carefully sketch a graph of the growth rate of this deer population as a function of population.

   (b) Determine a discrete dynamical system to model this deer population.

   (c) Determine the maximum interval of stability for this deer population.

7. Suppose that a man-made lake is stocked with 200 fish and that the fish population then grows logistically with an intrinsic growth rate of 8.5% and a carrying capacity of 800.

   (a) Carefully sketch a graph of the growth rate of this fish population as a function of population.

   (b) Determine a discrete dynamical system to model this fish population.

   (c) Determine the maximum interval of stability for this fish population.

8. There are currently 100 rabbits, but the population is expected to grow exponentially by 5% each year from now on. Sketch the graphs described. I will mainly be looking at the shape of your graph along with the values of any intercepts being clearly marked.

   (a) Sketch a graph of growth rate as a function of population.

(b) Sketch a graph of population as a function of time.

(c) Sketch a graph of yearly growth as a function of population.

9. Suppose that a population of wild pigs grows logistically with an intrinsic growth rate of 20% and a carrying capacity of 125. Sketch the graphs described. I will mainly be looking at the shape of your graph along with the values of any intercepts being clearly marked.

   (a) Sketch a graph of growth rate as a function of population.

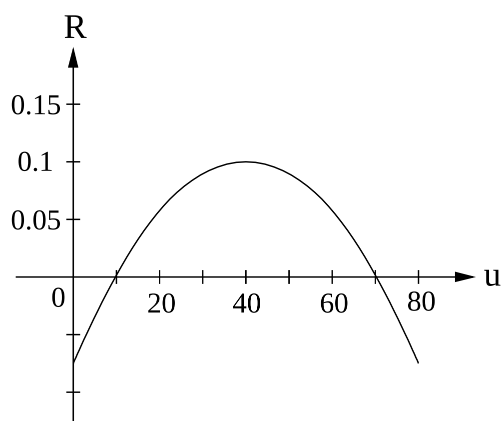
   (b) Sketch a graph of population as a function of time.

   (c) Sketch a graph of yearly growth as a function of population.

10. A population can be modeled by the following discrete dynamical system

$$u(n) = u(n-1) + R \cdot u(n-1)$$

where R is a function of the population u and is shown in the following graph.



   (a) Determine the intrinsic growth rate for this population?

   (b) Find all 3 equilibrium values for this population.

   (c) Sketch a rough graph of the population as a function of time, being sure to show each equilibrium value clearly and being sure to show what happens to any initial populations which are above or below each positive equilibrium value.

   (d) Determine the minimum viable population.

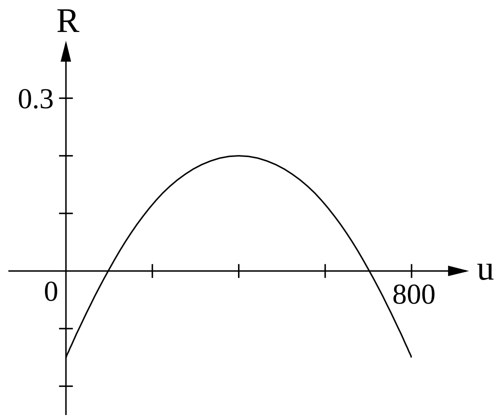
   (e) Find a formula for R as a function of u given that its graph is a parabola.

   (f) If $u(0) = 20$, then what is the value of $u(8)$?

11. A population can be modeled by the following discrete dynamical system

$$u(n) = u(n-1) + R \cdot u(n-1)$$

where R is a function of the population u and is shown in the following graph.



(a) Determine the intrinsic growth rate for this population?

(b) Find all 3 equilibrium values for this population.

(c) Sketch a rough graph of the population as a function of time, being sure to show each equilibrium value clearly and being sure to show what happens to any initial populations which are above or below each positive equilibrium value.

(d) Determine the minimum viable population.

(e) Find a formula for R as a function of u given that its graph is a parabola.

(f) If $u(0) = 200$, then what is the value of $u(10)$?

12. Suppose a certain chemical is eliminated from the body by the kidneys and the liver. Let $u(n)$ represent the amount of this chemical in a person's bloodstream after n days. Assume that each day, the kidneys remove 15% of the chemical from the blood. Also assume that each day, the fraction of the chemical that is broken down by enzymes from the liver is given by

$$\frac{4}{6 + u(n-1)}$$

Finally, assume that each day, the person takes a dose of 100 mg of this chemical. Develop a dynamical system for $u(n)$. You do not need an initial value.

13. The elimination rate r for alcohol in a person's bloodstream can be modeled using

$$r = \frac{b}{c + a}$$

where a is the number of grams of alcohol already in the bloodstream. Suppose a person eliminates one half of the alcohol in his bloodstream in an hour when he consumes 21 grams of alcohol, but only eliminates one third of the alcohol in an hour when he consumes 33 grams of alcohol. Determine the values of b and c in the elimination rate for this person.

14. Suppose the metabolism of some person is such that the discrete dynamical system modeling the elimination of alcohol is

$$a(n) = a(n-1) - \frac{10a(n-1)}{4 + a(n-1)} + d$$

where $a(n)$ is the amount of alcohol (in grams) in the person's bloodstream after n hours of drinking d grams of alcohol per hour.

Suppose this person's weight is such that 40 grams of alcohol in the bloodstream represents a blood alcohol level of 0.08 (the amount in South Carolina for a DWI conviction).

If this person drinks 3 cans of beer each hour for 3 hours, but then stops drinking for the next 3 hours, will his blood alcohol level fall below 40 grams? To justify your answer, you will need to fill in the remaining entries in the table below. Recall that each can of beer contains about 14 grams of alcohol.

Hint: Think carefully about the modifications needed to obtain the last 3 entries when he is no longer drinking.

n	a(n)
0	0
1	42.0
2	
3	
4	
5	
6	

15. For Beth's metabolism, the dynamical system modeling her elimination of alcohol is

$$a(n) = a(n-1) - \frac{9.5a(n-1)}{4.5 + a(n-1)} + d$$

where $a(n)$ is the amount of alcohol (in grams) in her bloodstream after n hours of drinking d grams of alcohol per hour. For Beth's weight, 37 grams of alcohol in the bloodstream represents a blood alcohol level of 0.08. Correct to one place after the decimal point, what is the largest number of grams of alcohol Mary can drink per hour at a 3-hour party if she wants to stay below that blood alcohol level of 0.08? Begin with $a(0) = 0$.

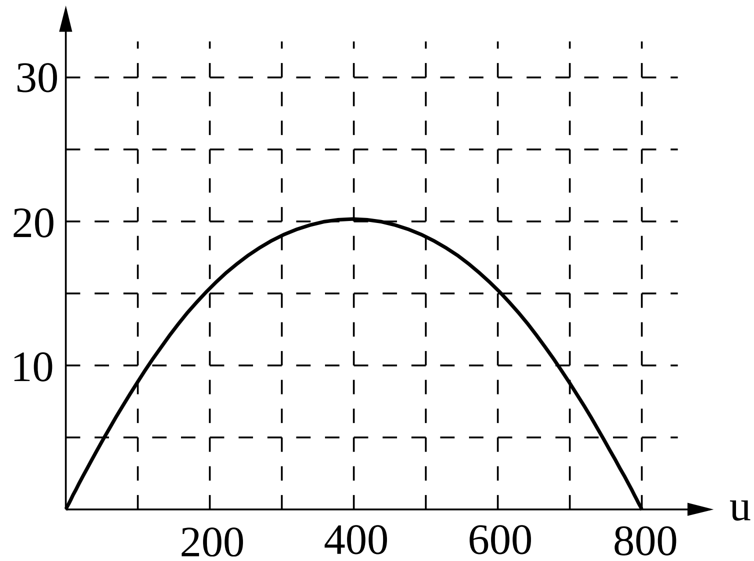
16. Suppose the metabolism of some person is such that the dynamical system modeling the elimination of alcohol is given by

$$a(n) = a(n-1) - \frac{9a(n-1)}{4.2 + a(n-1)} + d$$

where $a(n)$ is the amount of alcohol (in grams) in the bloodstream after n hours of drinking d grams of alcohol per hour. Compute the equilibrium amount of alcohol in the bloodstream and give a practical interpretation under the following circumstances.

(a) The person drinks 7 grams of alcohol per hour.
(b) The person drinks 8 grams of alcohol per hour.
(c) The person drinks 8.9 grams of alcohol per hour.
(d) The person drinks 9 grams of alcohol per hour.

17. The graph of the function $g = ru$, which gives the growth of a species in a year in terms of the population size, is seen in the following diagram.



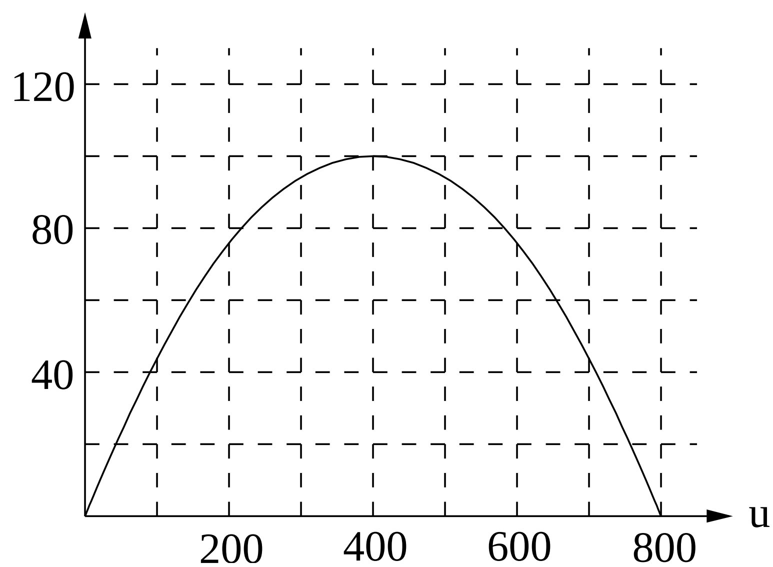
(a) Estimate the stable equilibrium population if there is a constant yearly harvest of 15.

(b) Estimate the minimum viable population if there is a constant yearly harvest of 15.

(c) Use this graph to estimate the maximum constant sustainable harvest and the equilibrium population size for this harvest.

(d) Approximate the percent of the population that should be harvested each year to maximize the sustainable harvest.

18. The graph of the function $g = ru$, which gives the growth of a species in a year in terms of the population size, is seen in the following diagram.



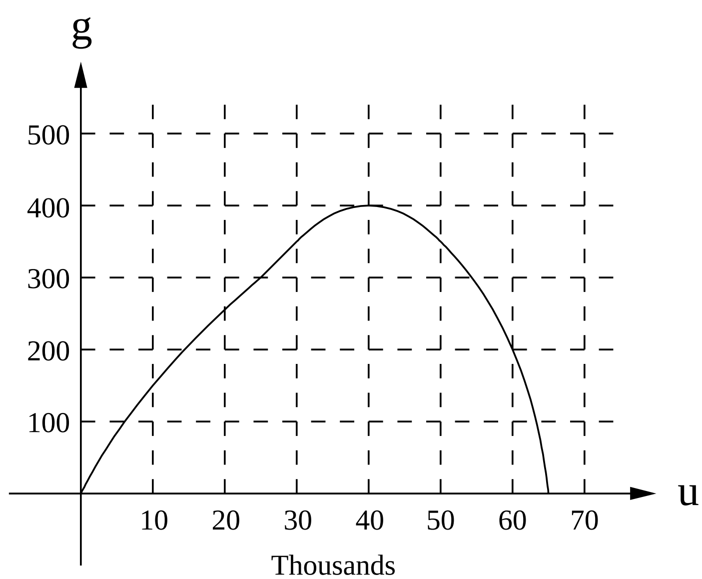
(a) Estimate the stable equilibrium population if there is a constant yearly harvest of 60.

(b) Estimate the minimum viable population if there is a constant yearly harvest of 60.

(c) Use this graph to estimate the maximum constant sustainable harvest and the equilibrium population size for this harvest.

(d) Approximate the percent of the population that should be harvested each year to maximize the sustainable harvest.

19. The graph of the function $g = ru$, which gives the growth of a species in a year in terms of the population size, is seen in the following diagram. The harvesting strategy is to harvest 100 plus an additional 0.5% of the population each year.

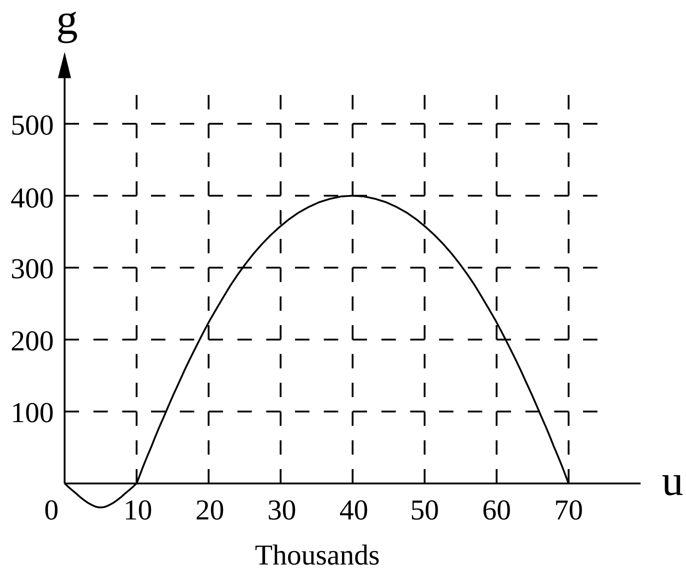


(a) Estimate the stable equilibrium population.

(b) Estimate the minimum viable population.

(c) Estimate the eventual yearly harvest as long as the current population is above the minimum viable population.

20. The graph of the function $g = ru$, which gives the growth of a species in a year in terms of the population size, is seen in the following diagram.



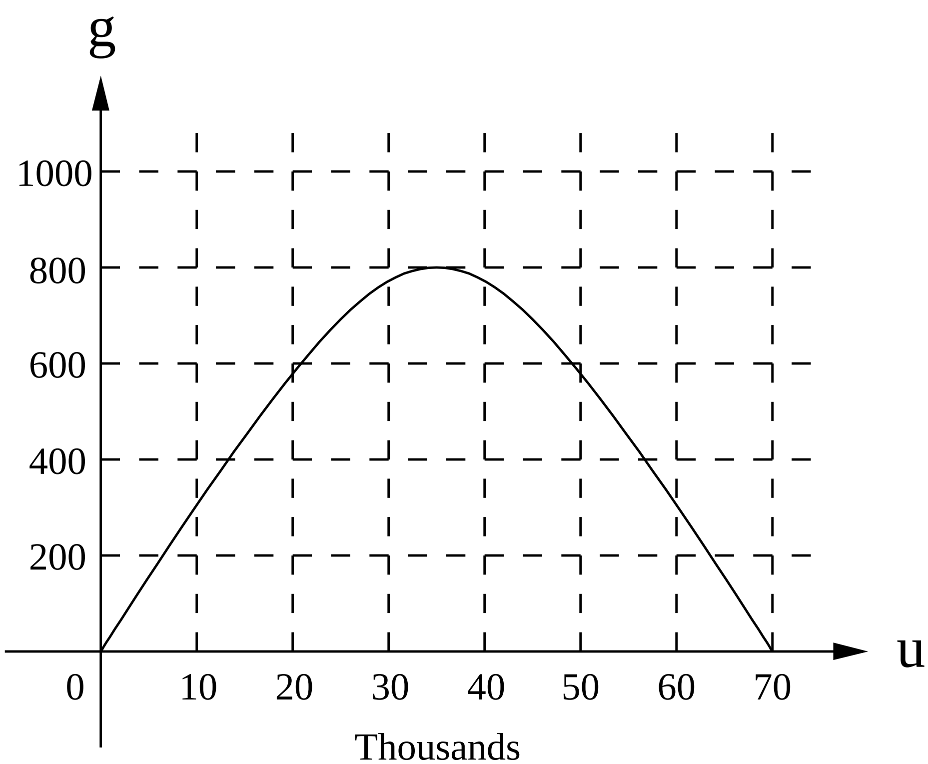
(a) Estimate the stable equilibrium population if there is a constant yearly harvest of 300.

(b) Estimate the minimum viable population if there is a constant yearly harvest of 300.

(c) Estimate the maximum constant sustainable harvest and the equilibrium population size for this harvest.

(d) Approximate the percent of the population that should be harvested each year to maximize the sustainable harvest.

21. The graph of the function $g = ru$, which gives the growth of a species in a year in terms of the population size, is seen in the following diagram. Each year the harvesting strategy is to harvest 1% of the population in excess of $10,000$.



(a) Determine a formula for the yearly harvest, h, as a function of the population u.

(b) Estimate the stable equilibrium population.

(c) Estimate the eventual yearly harvest.