Math 500, Homework 9

(1) Compute the Galois group of $x^3 - 3x + 17 \in \mathbb{Q}[x]$.

(2) Compute the Galois group of $x^{10} + x^5 + 1 \in \mathbb{Q}[x]$.

(3) Compute the Galois group of $x^4 - 5$ over $\mathbb{Q}$ and over $\mathbb{Q}(\sqrt{5})$.

(4) Which of the following are solvable by radicals over $\mathbb{Q}$? $f(x) = x^5 - 3x + 2$, $g(x) = x^5 - 13x + 13$, $h(x) = x^5 - 4x - 2$. [Note that any subgroup of S_4 is solvable, cf. Rotman's *Galois Theory*, Theorem G.34.]

(5) Compute the Galois group of $f(x) = x^3 - 12x + 21$ over $\mathbb{Q}$ and over $\mathbb{R}$.

(6) Compute the Galois group of $x^5 - 1$ over $\mathbb{Q}$.

(7) Compute the Galois group of $x^4 + x^2 - 6 \in \mathbb{Q}[x]$.

(8) Compute the Galois group of $x^4 + x^2 + x + 1 \in \mathbb{Q}[x]$.