Correspondence Theorem

Let G and H be groups, and let $\varphi : G \to H$ be an epimorphism having kernel $N = \ker(\varphi)$. Then there is a bijective correspondence given by φ between subgroups of G that contain N and subgroups of H: if $N \subseteq K$, K a subgroup of G, this correspondence sends K to $\varphi(K)$, while if $L \subseteq H$ is a subgroup, the correspondence sends L to $\varphi^{-1}(L)$.

Moreover, under this correspondence, if K_1, K_2 are subgroups of G containing N, then

(1) $K_2 \subseteq K_1$ iff $\varphi(K_2) \subseteq \varphi(K_1)$

(2) K_2 is normal in K_1 iff $\varphi(K_2)$ is normal in $\varphi(K_1)$, and in this case

$$K_1/K_2 \cong \varphi(K_1)/\varphi(K_2).$$