

Math 428, Homework 1, due Friday, August 25

Problem 1. Describe the maximal ideals in $\mathbb{C}[x]$; describe the prime ideals in $\mathbb{C}[x]$. Justify your answer.

Problem 2. Describe the prime ideals in $\mathbb{R}[x]$. Justify your answer.

Problem 3. Describe the prime ideals in $\mathbb{C}[x]/(x^2 - 1)$. Justify your answer.

Problem 4. Describe the prime ideals in \mathbb{Z} . Justify your answer.

Problem 5. Let M denote the ring of 2×2 complex matrices with usual matrix addition and multiplication (recall that this ring is *not* commutative). Let

$$X = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \quad Y = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Define functions $\phi : \mathbb{C}[x] \rightarrow M$ and $\psi : \mathbb{C}[y] \rightarrow M$ by $\phi(f) = f(X)$ and $\psi(g) = g(Y)$. Prove that ϕ and ψ are ring homomorphisms. Compute the kernels of ϕ and ψ . What are the prime ideals of the images of ϕ and ψ ?