(1) Consider the field extension

$$\mathbb{Q}(\sqrt{2})(\sqrt{3}) \text{ of } \mathbb{Q}.$$

(a) Prove that $[\mathbb{Q}(\sqrt{2})(\sqrt{3}) : \mathbb{Q}] = 4$.

(b) Find an irreducible polynomial of degree 4, $p(x) \in \mathbb{Q}[x]$, such that $p(\sqrt{2}+\sqrt{3}) = 0$.

(c) Conclude that

$$\mathbb{Q}(\sqrt{2})(\sqrt{3}) = \mathbb{Q}(\sqrt{2}+\sqrt{3}).$$

(2) Let $\zeta_n = e^{2\pi i/n}$.

(a) What is

$$[\mathbb{Q}(\zeta_5) : \mathbb{Q}] ?$$

(b) What is

$$[\mathbb{Q}(\zeta_6) : \mathbb{Q}] ?$$

Justify!

(3) A linear fractional transformation (LFT) of $\mathbb{C}$ is a function

$$f(z) = \frac{az+b}{cz+d} \quad \text{where}$$

$$a, b, c, d \in \mathbb{C} \text{ and } ad-bc \neq 0.$$

[Its domain is $\{z \mid cz+d \neq 0\}$.].

(a) How are LFTs related to projective transformations of $\mathbb{P}^1_{\mathbb{C}}$?

(b) What is the significance of the point z (if there is one) at which $f(z)$ is not defined?

(4). Do problems 3-2, 3-3, 3-5, from Fulton. Also, do problem 3-17 for curves A and C, for curves C and D, and for curves C and E.

(5) (a) Prove that if $f_d, f_{d+1} \in \mathbb{C}[X,Y]$ are homogeneous polynomials of degrees d and $d+1$ respectively ($d \geq 1$) and $f_d + f_{d+1}$ is irreducible then the plane curve $V(f_d + f_{d+1})$ admits a rational parametrization.

(b) Show that, after a change of variables, this proves that the circle $V(x^2+y^2-1)$ can be rationally parametrized.