Rough Proof of Torelli   Pick basepoint $b \in X$.

(1) Class Θ is represented by divisor

$$W_{g-1} = \mathrm{Im}\left(\mathrm{div} : \mathrm{Sym}^{g-1} X \longrightarrow \mathrm{Jac}(X)\right)$$

$$(p_1, \dots, p_{g-1}) \longmapsto \sum p_i - (g-1)b.$$

[Better: replace $\mathrm{Jac}(X)$ by $\mathrm{Pic}^{g-1}(X)$!]

(2) <u>Gauss map</u> Given $Y \subset Z$ nonsingular, with $TZ \cong Z \times \mathbb{C}^{\dim Z}$, get

$$Y \xrightarrow{G} \mathrm{Gr}_{\dim Y}(\mathbb{C}^{\dim Z}) \text{ by}$$

$$y \longmapsto T_y \subset T_y Z = \mathbb{C}^{\dim Z}.$$

<u>Ex 1</u> $X \xrightarrow{\text{Abel-Jacobi}} \mathrm{Jac}(X) = H^{0,1}(X)/H^1(X,\mathbb{Z})$

$$\cong H^{1,0}(X)^* / H_1(X,\mathbb{Z})$$

$$x \longmapsto \left[\omega \longmapsto \int_b^x \omega\right].$$

<u>Note</u> Depends on path $b \rightsquigarrow x$, but difference between paths is an integral over a <u>loop</u> $\in H_1(X,\mathbb{Z})$.

Gauss map is, in terms of basis $\omega_1, \dots, \omega_g$ of $H^{1,0}(X)$,

$$G_{AJ}(z) = \frac{\partial}{\partial z}\left[\int_b^z \omega_1, \dots, \int_b^z \omega_g\right] = \left[\frac{\omega_1(z)}{dz}, \dots, \frac{\omega_g(z)}{dz}\right]$$

(point in $\mathbb{P}^{g-1}$ doesn't depend on dz).

<u>Def</u> The map $X \xrightarrow{\Phi_K} \mathbb{P}^{g-1}$

$$z \longmapsto \left[\frac{\omega_1(z)}{dz} ; \dots ; \frac{\omega_g(z)}{dz}\right]$$ is the

<u>canonical map</u> of X.

<u>From now</u> Assume X <u>nonhyperelliptic</u>, so Φ_K is an embedding.

<u>Ex 2</u> $W_{g-1}^{\text{nonsing}} \subset \mathrm{Pic}^{g-1} X = \mathrm{Jac}\, X$.

<u>Geometric Fact</u> $[p_1 + \dots + p_{g-1}] \in W_{g-1}^{\text{nonsing}}$. Then

$$T_{[p_1+\dots+p_{g-1}]} W_{g-1}^{\text{nonsing}} \subset T_{[p_1+\dots+p_{g-1}]} \mathrm{Jac}(X) = H^{0,1}(X)$$

is the hyperplane spanned by $p_1, \dots, p_{g-1}$.

<u>Branch Locus</u> of Gauss map $G : W_{g-1}^{ns} \longrightarrow \mathbb{P}(H^{0,1}(X)^*)$:

locus "in target" where matrix of partial derivs. drops rank.

[Both base, target dim g, so branch locus is a hypersurface]

How to identify it: s'pose $p_1, \dots, p_{g-1} \in X$,

$T_{p_i} \Phi_K(X) \subset \mathrm{span}\{p_1, \dots, p_{g-1}\} = H$.

Then "small movement of p_i doesn't change $H = G([p_1 + \dots + p_{g-1}])$",

so $G([p_1 + \dots + p_{g-1}]) \in$ branch locus.

Conversely, suppose $H = \text{span}\{p_1, \dots, p_{g-1}\}$ not tangent to X.

Then $H \cap X$ consists of $2g-2$ points (canonical linear series has deg $2g-2$), so $\exists$ $\binom{2g-2}{g-1}$ points of $\text{Sym}^{g-1} X$ (i.e. divisors of deg $g-1$ on X) whose image under G is H.

Calculation shows This is number of sheets of the covering G.

So such a point can't be a branch point.

Conclusion

(*) $$\text{Branch locus}(G) = \{ H \mid T_p \Phi_K(X) \subset H \text{ for some } p \in \Phi_K(X) \cap H \}$$

Rmk This is a little careless: more careful reasoning shows really closure $\overline{Br(G)}$ is this set.

Final Step RHS of (*) reconstructs X.

This is a little bit more, but not hard to believe: just saying set of tangent hyperplanes through points of C reconstructs C.