Answers due 9/25

HW # 3

2.8: 6) $\begin{pmatrix} 1 & 4 & 5 & -4 \\ -2 & -7 & -8 & 10 \\ 4 & 9 & 6 & -7 \\ 3 & 7 & 8 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 4 & 5 & -4 \\ 0 & 1 & 2 & 2 \\ 0 & -7 & -14 & 9 \\ 0 & -5 & -10 & 7 \end{pmatrix} \sim$

$$\begin{pmatrix} 1 & 4 & 5 & -4 \\ 0 & 1 & 2 & 2 \\ \boxed{0 \quad 0 \quad 0 \quad 1} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

v is not in subspace $\{v_1, v_2, v_3\}$

$$0 \neq 1$$

Ch 2
Supp. Ex. 11) a. $\begin{pmatrix} c_0 \\ \vdots \\ c_{n-1} \end{pmatrix} = \text{row}_i(V)c = \text{row}_i(Vc) = \text{row}_i(y)$
$= y_i$
b/c $Vc = y$

b. c are the coefficients of a polynomial whose value is zero at $x_1 \ldots x_n$ if $Vc = 0$. But, a nonzero polynomial w/ $n-1$ degree can't have n zeros, so c must all be zeros. So, V is linearly independent.

c. So, V is linear independent and columns span R^n. For every y_i in R^n there is a vector such that $Vc = y$. The polynomial whose coefficients are listed in c is an interpolating polynomial for $(x_1, y_1) \ldots (x_n, y_n)$

4.2: 24) $\begin{pmatrix} -8 & -2 & -9 & 2 \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 & 2 & -1 \\ 0 & -2 & -1 & -2 \\ 0 & 4 & 2 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & -1/2 \\ 0 & 1 & 1/2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\rightarrow$ w is in Col A

$$Aw = \begin{pmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\rightarrow$ w is in Nul A

4.3: 14) Basis for Col A is

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} -5 \\ -5 \\ 0 \\ -5 \end{pmatrix}, \begin{pmatrix} -3 \\ 2 \\ 5 \\ -2 \end{pmatrix} \right\}$$

$$\begin{pmatrix} 1 & 2 & 0 & 4 & 5 & 0 \\ 0 & 0 & 5 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 & -9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$x_5 = 0$
$x_4 = \text{free}$
$x_3 = \dfrac{-8x_5 + 7x_4}{5} = \dfrac{7}{5}x_4$

$x_2 = \text{free}$
$x_1 = -5x_5 - 4x_4 - 2x_2 =$
$= -4x_4 - 2x_2$

$$X = x_2 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -4 \\ 0 \\ 7/5 \\ 1 \\ 0 \end{pmatrix}$$

Basis for Nul A is: $\left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ 7/5 \\ 1 \\ 0 \end{pmatrix} \right\}$

20) So, $\{v_1, v_2\}$, $\{v_2, v_3\}$, and $\{v_1, v_3\}$ all span H. None of the vectors are a multiple of another vector. So, the 3 sets are linearly independent and each form a basis for H.