

### Mathematics 511, Homework 3

- (1) Describe the points and the sheaf of functions on each of the schemes  $\text{Spec } \mathbb{C}[x]/(x^2)$ ;  $\text{Spec } \mathbb{C}[x]/(x^3 - x^2)$ ;  $\text{Spec } \mathbb{R}[x]/(x^2 + 1)$ .
- (2) Let  $X$  be a topological space with three points  $\{p, q_1, q_2\}$ . Topologize  $X$  by making  $X_1 = \{p, q_1\}$  and  $X_2 = \{p, q_2\}$  into open sets (as well as  $X, \emptyset, \{p\}$ ). Define a presheaf of rings  $\mathcal{O}$  on  $X$  by  $\mathcal{O}(X) = \mathcal{O}(X_1) = \mathcal{O}(X_2) = \mathbb{C}[x]_{(x)}$ ,  $\mathcal{O}(\{p\}) = \mathbb{C}(x)$ , with the “obvious” restriction maps. Show that this presheaf is a sheaf. Prove that  $(X, \mathcal{O})$  is a scheme. Prove that it is *not* an affine scheme.
- (3) Do problems 2.1-2.4, 2.12, 2.13, of Chapter II of Hartshorne, not to be turned in.
- (4) Do problems 2.5-2.9 of Chapter II of Hartshorne, to be turned in.