Problem 3 $\quad |\vec{v}| = 2, \quad |\vec{w}| = 3.$

$$|\vec{v} \times \vec{w}| = |\vec{v}||\vec{w}| \sin(\theta)$$
$$= 2 \cdot 3 \cdot \sin\left(\frac{\pi}{4}\right)$$
$$= \frac{2 \cdot 3}{\sqrt{2}} = \boxed{3\sqrt{2}}$$


Problem 4 $\quad p = (2,5,1), \quad q = (3,0,-7).$

(1) $\vec{pq} = \langle 3, 0, -7\rangle - \langle 2, 5, 1\rangle$
$$= \langle 1, -5, -8\rangle.$$

(2) line through $p$ and $q$ is
$$\vec{r}(t) = \langle 2,5,1\rangle + t\langle 1,-5,-8\rangle$$
$$= \langle 2+t, 5-5t, 1-8t\rangle.$$

(3) area of parallelogram is

$|\vec{Op} \times \vec{Oq}|$ = abs. value of

$$\langle 2,5,1\rangle \times \langle 3,0,-7\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 5 & 1 \\ 3 & 0 & -7 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} 5 & 1 \\ 0 & -7 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 1 \\ 3 & -7 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 5 \\ 3 & 0 \end{vmatrix}$$

$= \langle -35, 17, -15\rangle$. length is

$$\sqrt{35^2 + 17^2 + 15^2}$$

$$= \sqrt{1225 + 289 + 225}$$

$$= \sqrt{1739}$$

(4) $\vec{pq} = \langle 1, -5, -8\rangle$ as above

$\vec{pr} = \langle 1,4,4\rangle - \langle 2,5,1\rangle$

$= \langle -1, -1, 3\rangle$.

$$\vec{pq} \times \vec{pr} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -5 & -8 \\ -1 & -1 & 3 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} -5 & -8 \\ -1 & 3 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & -8 \\ -1 & 3 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -5 \\ -1 & -1 \end{vmatrix}$$

$$= \langle -23, 5, -6 \rangle.$$

vector eq.

$$0 = \langle -23, 5, -6 \rangle \cdot \left( \langle x, y, z \rangle - \langle 2, 5, 1 \rangle \right)$$

scalar eq.

$$0 = -23(x-2) + 5(y-5) - 6(z-1)$$

$$= -23x + 5y - 6z + 46 - 25 + 6$$

or

$$0 = -23x + 5y - 6z + 27$$

(5) volume of parallelepiped is abs. value of

$$\begin{vmatrix} 2 & 5 & 1 \\ 3 & 0 & -7 \\ 1 & 4 & 4 \end{vmatrix} = 2\begin{vmatrix} 0 & -7 \\ 4 & 4 \end{vmatrix} - 5\begin{vmatrix} 3 & -7 \\ 1 & 4 \end{vmatrix} + \begin{vmatrix} 3 & 0 \\ 1 & 4 \end{vmatrix}$$

$$= 2 \cdot 28 - 5(19) + 12 = 56 - 95 + 12$$

$$= -27 \quad \text{so} \quad \boxed{27}$$

<u>Problem 5</u>

(1) $$\text{comp}_{\vec{a}}(\vec{b}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

$$= \frac{\langle 2,1,0\rangle \cdot \langle 1,1,-1\rangle}{|\langle 2,1,0\rangle|} = \frac{3}{\sqrt{5}}$$

(2) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$. So,

$$\cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{3}{|\vec{a}||\vec{b}|}$$

$$= \frac{3}{(\sqrt{5})\sqrt{3}} = \boxed{\frac{3}{\sqrt{15}}}$$

(3) $$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 0 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$= \langle -1, 2, 1 \rangle$. length is

$\sqrt{1+4+1} = \sqrt{6}$.

So, $\left\langle -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$.

Problem 6

$$\vec{r}\,'(t) = \left\langle -4\sin(t),\ -4\cos(t),\ 3 \right\rangle.$$

$$|\vec{r}\,'(t)| = \sqrt{16\sin^2 t + 16\cos^2 t + 9}$$

$$= \sqrt{16+9} = 5.$$

$$\vec{T}(t) = \left\langle -\frac{4}{5}\sin(t),\ -\frac{4}{5}\cos(t),\ \frac{3}{5} \right\rangle.$$

(1) $$\vec{T}\left(\frac{\pi}{4}\right) = \left\langle -\frac{4}{5\sqrt{2}},\ \frac{-4}{5\sqrt{2}},\ \frac{3}{5} \right\rangle$$

$$\frac{d\vec{T}}{dt} = \left\langle -\frac{4}{5}\cos(t),\ \frac{4}{5}\sin(t),\ 0 \right\rangle$$

$$\frac{d\vec{T}}{dt}\left(\frac{\pi}{4}\right) = \left\langle -\frac{4}{5\sqrt{2}},\ \frac{4}{5\sqrt{2}},\ 0 \right\rangle.$$

$$\left|\frac{d\vec{T}}{dt}\left(\frac{\pi}{4}\right)\right| = \sqrt{\frac{16}{50}+\frac{16}{50}} = \sqrt{\frac{16}{25}} = \frac{4}{5}.$$

So,

(2) $$\frac{\frac{d\vec{T}}{dt}\left(\frac{\pi}{4}\right)}{\left|\frac{d\vec{T}}{dt}\left(\frac{\pi}{4}\right)\right|} = \boxed{\left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\rangle.}$$

is unit normal.

(3) Normal comp. of accel. is

$$a_N = \frac{|\vec{v}\times\vec{a}|}{|\vec{v}|}. \qquad \vec{r}''(t) = \langle -4\cos t, 4\sin t, 0\rangle.$$

$$\vec{r}'\left(\frac{\pi}{4}\right) = \left\langle -\frac{4}{\sqrt{2}}, -\frac{4}{\sqrt{2}}, 3\right\rangle. = \vec{v}\left(\frac{\pi}{4}\right)$$

$$\vec{r}''\left(\frac{\pi}{4}\right) = \left\langle -\frac{4}{\sqrt{2}}, \frac{4}{\sqrt{2}}, 0\right\rangle. = \vec{a}\left(\frac{\pi}{4}\right).$$

So,

$$\vec{v} \times \vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\frac{4}{\sqrt{2}} & -\frac{4}{\sqrt{2}} & 3 \\ -\frac{4}{\sqrt{2}} & \frac{4}{\sqrt{2}} & 0 \end{vmatrix}$$

$$= \vec{i}\left(\frac{-12}{\sqrt{2}}\right) - \vec{j}\left(\frac{-12}{\sqrt{2}}\right) + \vec{k}\left(-8-8\right)$$

$$= \left\langle -\frac{12}{\sqrt{2}}, \frac{12}{\sqrt{2}}, -16 \right\rangle.$$

$$|\vec{v} \times \vec{a}| = \sqrt{\left(\frac{-12}{\sqrt{2}}\right)^2 + \left(\frac{12}{\sqrt{2}}\right)^2 + (-16)^2}$$

$$= \sqrt{144 + 256} = \sqrt{400} = 20.$$

Also, $|\vec{v}| = 5$ (already computed!).

So, $\boxed{a_N = \frac{20}{5} = 4}$

## Problem 7

$$\vec{v}(t) = \vec{v}(0) + \int_0^t \vec{a}(u)\,du$$

$$= \langle 0, 10, 0 \rangle + \left\langle \int_0^t u\,du,\ \int_0^t u^2\,du,\ \int_0^t u^2\,du \right\rangle$$

$$= \left\langle \frac{t^2}{2},\ \frac{t^3}{3} + 10,\ \frac{t^3}{3} \right\rangle.$$

$$\vec{r}(t) = \vec{r}(0) + \int_0^t \vec{v}(u)\,du$$

$$= \langle 10, 0, 0 \rangle + \left\langle \int_0^t \frac{u^2}{2}\,du,\ \int_0^t \left(\frac{u^3}{3} + 10\right)du,\ \int_0^t \frac{u^3}{3}\,du \right\rangle$$

$$= \left\langle 10 + \frac{t^3}{6},\ \frac{t^4}{12} + 10t,\ \frac{t^4}{12} \right\rangle.$$

Problem 8

$$v(t) = |\vec{v}(t)| = \sqrt{\tan^2(2t) + 1}$$

$$= \sqrt{\sec^2(2t)}$$

$$= \frac{1}{\cos(2t)}.$$

$$\vec{T}(t) = \frac{\vec{v}(t)}{v(t)} = \langle \tan(2t)\cdot\cos(2t), 0, -\cos(2t) \rangle$$

$$= \langle \sin(2t), 0, -\cos(2t) \rangle$$

$$\frac{d\vec{T}}{dt} = \langle 2\cos(2t), 0, 2\sin(2t) \rangle.$$

$$\left|\frac{d\vec{T}}{dt}\right| = \sqrt{4\cos^2 2t + 4\sin^2 2t} = \sqrt{4} = 2.$$

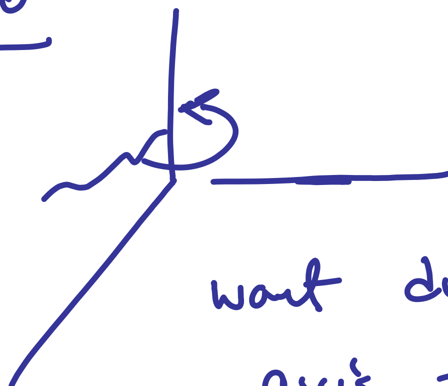
$$K = \left|\frac{d\vec{T}}{dt}\right| \cdot \frac{1}{v} = 2\cos(2t).$$

<u>Remark</u> It should have read $t = \frac{\pi}{8}$ to get a sensible answer. At $t = \frac{\pi}{4}$, $\tan(2t)$ is undefined, and the above calculations don't really make sense. The expression for curvature is correct whenever $\vec{r}(t)$, $\vec{T}(t)$ and $\frac{d\vec{T}}{ds}$ are defined!

<u>Problem 9</u>

$$(x-1)^2 + (y-3)^2 + (z+7)^2 = 16.$$

Problem 10



want dist. fr. pt. to $z$ axis to satisfy

$$(\text{dist})^2 + 3z^3 = 1.$$

so,

$$\left(\sqrt{x^2+y^2}\right)^2 + 3z^3 = 1, \quad \text{or}$$

$$\boxed{x^2+y^2+3z^3=1}$$