Math 241, Practice Midterm 1

This gives some reasonable practice problems. The actual exam will not look precisely like this!

Problem 1. State the "Interpretation of Dot Product" Theorem.

Note: equally good review problems:

- State the "Test for Perpendicular Vectors" Corollary.
- State the "Perpendicularity of the Cross Product" Theorem.

Problem 2. Give the formula for the dot product $\mathbf{v} \cdot \mathbf{w}$ of two vectors

$$\mathbf{v} = \langle v_1, v_2, v_3 \rangle \qquad \text{and} \qquad \mathbf{w} = \langle w_1, w_2, w_3 \rangle.$$

Problem 3. Suppose $\mathbf{v}$ and $\mathbf{w}$ are vectors of lengths 2 and 3, respectively, and such that the angle between them is $\pi/4$. What is $|\mathbf{v} \times \mathbf{w}|$?

Problem 4. Given the points $p = (2, 5, 1)$ and $q = (3, 0, -7)$, compute:

(1) The vector $\overrightarrow{pq}$.
(2) A parametric equation for the line through the points p and q.
(3) The area of the parallelogram determined by the three points p, q, and $(0, 0, 0)$.
(4) A scalar equation for the plane through the points p, q, and $r = (1, 4, 4)$.
(5) The volume of the parallelepiped determined by $\overrightarrow{op}$, $\overrightarrow{oq}$, and $\overrightarrow{or}$ (where o denotes the origin $o = (0, 0, 0)$).

Problem 5. Given the vectors $\mathbf{a} = \langle 2, 1, 0 \rangle$ and $\mathbf{b} = \langle 1, 1, -1 \rangle$, compute:

(1) The component of $\mathbf{b}$ in the direction of $\mathbf{a}$.
(2) The **cosine** of the angle between $\mathbf{a}$ and $\mathbf{b}$.
(3) A unit vector that is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

Problem 6. Given the vector-valued function $\mathbf{r}(t) = \langle 4\cos(t), -4\sin(t), 3t + 2 \rangle$, compute:

(1) The unit tangent vector at $t = \pi/4$.
(2) The unit normal vector at $t = \pi/4$.
(3) The normal component of acceleration at $t = \pi/4$.

Problem 7. Suppose a particle moves in space with position function $\mathbf{r}(t)$. Suppose the acceleration $\mathbf{a}(t)$ is given by

$$\mathbf{a}(t) = \langle t, t^2, t^2 \rangle.$$

Suppose also that the initial position is $\mathbf{r}(0) = \langle 10, 0, 0 \rangle$, and the initial velocity is $\mathbf{v}(0) = \langle 0, 10, 0 \rangle$. Compute the function $\mathbf{r}(t)$.

Problem 8. Suppose a vector-valued function $\mathbf{r}(t)$ has velocity given by

$$\mathbf{v}(t) = \langle \tan(2t), 0, -1 \rangle.$$

Compute the curvature κ of the corresponding curve in space at time $t = \pi/4$.

Problem 9. Give the equation of the sphere in 3-dimensional space with center at $(1, 3, -7)$ and radius 4.

Problem 10. Give the equation of the surface of revolution obtained by taking the curve in the xz plane defined by $x^2 + 3z^3 = 1$ and revolving it around the z-axis.