Math 241, Suggested Midterm 3 Review Problems

Problem 1.

(1) Express $(x, y, z) = (1, 1, -\sqrt{2})$ in spherical coordinates (ρ, ϕ, θ).

(2) Express the set

$$\{(x, y, z) \mid x \geq 0 \text{ and } y = -x\}$$

using an equation in cylindrical coordinates.

Problem 2. Compute the integral:

$$\int_{-2}^{1} \int_{2}^{4} x^2 y^3 \, dy \, dx.$$

Problem 3. Compute the volume of the solid that lies inside the cylinder

$$y^2 + z^2 = 4,$$

"in front of" the plane $x = 0$ (that is, whose points satisfy $x \geq 0$), and "behind" the plane $x = y + 2$. [Draw the picture to figure out what this means.]

Problem 4. Reverse the order of integration and compute the double integral:

$$\int_{0}^{4} \int_{\sqrt{y}}^{2} \frac{y e^{x^2}}{x^3} \, dx \, dy.$$

Problem 5.

(1) Give the formulas for the mass and centroid of a lamina in the xy-plane that occupies a region R and has density $\delta(x, y)$.

(2) Find the centroid of the region in the xy-plane bounded by the x-axis and the parabola $y = 4 - x^2$ (that is, with density 1).

Problem 6.

(1) Give the formula for the polar moment of inertia of a plane lamina occupying a region R and with density $\delta(x, y)$.

(2) Compute the polar moment of inertia of the plane lamina that occupies the disk enclosed by the circle

$$x^2 + y^2 = 4$$

and with density function $\delta(x, y) = x^2$.

Problem 7. Let R denote the region in the first quadrant of the plane given by:

$$R = \{(x, y) \mid 1 - 2x^2 \leq y \leq 3 - 2x^2, \;\; -1 + 2x^2 \leq y \leq 2x^2, \;\; x \geq 0\}.$$

(1) Given the change of variables

$$u = y + 2x^2, \;\; v = y - 2x^2,$$

solve for x and y in terms of u and v to obtain a transformation

$$T(u, v) = (x(u, v), y(u, v)).$$

(2) Compute the Jacobian determinant $J_T(u, v)$ of $T(u, v)$.

(3) Express the integral

$$\iint_R (x^2 + 2x + y^2) \, dx \, dy$$

as an integral in the variables u and v.