

**Problem.** Compute the volume of the solid that lies inside the cylinder

$$x^2 + y^2 = 1,$$

above the plane  $z = -2$ , and below the surface  $z = xy + 4$ . Show your work.

$$\text{let } R = \{ (x, y) \mid x^2 + y^2 \leq 1 \}.$$

$$\text{vol} = \iint_R [xy + 4 - (-2)] dA = \iint_R [xy + 6] dA$$

$$= \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=1} [(r \cos \theta)(r \sin \theta) + 6] r dr d\theta$$

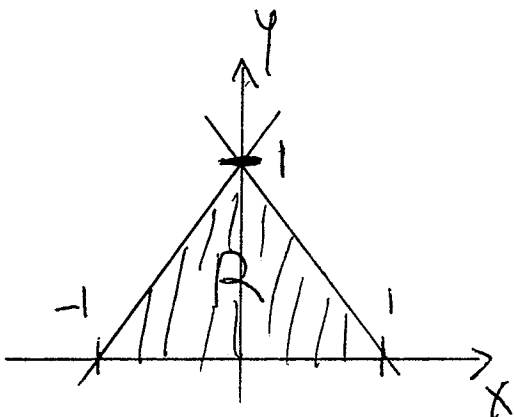
$$= \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=1} [r^3 \cos \theta \sin \theta + 6r] dr d\theta$$

$$= \int_0^{2\pi} \left[ \frac{r^4}{4} \cos \theta \sin \theta + 3r^2 \right] \Big|_{r=0}^{r=1} d\theta = \int_0^{2\pi} \left[ \frac{1}{4} \cos \theta \sin \theta + 3 \right] d\theta$$

$$= \frac{1}{8} \sin^2 \theta \Big|_{\theta=0}^{\theta=2\pi} + 3\theta \Big|_{\theta=0}^{\theta=2\pi}$$

$$= \boxed{6\pi}$$

**Problem.** Compute the volume of the solid that lies below the surface  $z = y^3 + \sqrt{y}$  and above the triangular region  $R$  in the  $xy$  plane depicted below:



$$R = \left\{ (x, y) \mid \begin{array}{l} y-1 \leq x \leq 1-y, \\ 0 \leq y \leq 1 \end{array} \right\}$$

Show your work.

$$\text{Volume} = \int_{y=0}^{y=1} \int_{x=y-1}^{x=1-y} (y^3 + \sqrt{y}) \, dx \, dy$$

$$= \int_0^1 (y^3 + \sqrt{y}) [(1-y) - (y-1)] \, dy$$

$$= \int_0^1 (y^3 + \sqrt{y})(2-2y) \, dy = \int_0^1 (2y^3 + 2\sqrt{y} - 2y^4 - 2y^{3/2}) \, dy$$

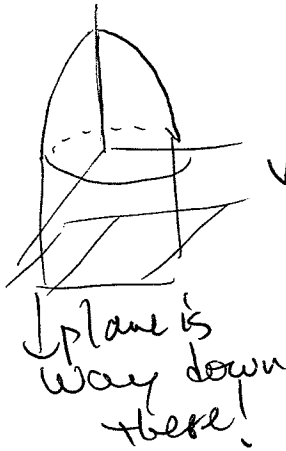
$$= \left( \frac{y^4}{2} + \frac{4}{3}y^{3/2} - \frac{2y^5}{5} - \frac{4}{5}y^{5/2} \right) \Big|_0^1$$

$$= \frac{1}{2} + \frac{4}{3} - \frac{2}{5} - \frac{4}{5} = \frac{15}{30} + \frac{40}{30} - \frac{12}{30} - \frac{24}{30} = \boxed{\frac{19}{30}}$$

**Problem.** Compute the volume of the solid that lies inside the cylinder

$$x^2 + y^2 = 25,$$

below the paraboloid  $z = 25 - x^2 - y^2$ , and above the plane  $z = x - 20$ . Show your work.



$$\text{vol} = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=5} [(25 - r^2) - (r \cos \theta - 20)] r dr d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=5} [25r - r^3 - r^2 \cos \theta + 20r] dr d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi} \left[ \frac{25r^2}{2} - \frac{r^4}{4} - \frac{r^3}{3} \cos \theta + 10r^2 \right]_0^5 d\theta$$

Use  $25^2 = 625$

$$= \int_{\theta=0}^{\theta=2\pi} \left[ \frac{625}{2} - \frac{625}{4} - \frac{125}{3} \cos \theta + 250 \right] d\theta$$

$$= \left[ \frac{625}{4} \theta - \frac{125}{3} \sin \theta + 250\theta \right]_{\theta=0}^{\theta=2\pi}$$

$$= \frac{625\pi}{2} + 500\pi = \boxed{\frac{1625\pi}{2}}$$