Math 241 §BL1

Lecture 11 Problems

(1) Let $F(x, y) = x^2 - xy^3 + 1$. Let $p = (1, 1, 1)$.

(a) Give parametrizations of the x-curve and y-curve (that is, the traces) on the surface $z = F(x, y)$ through the point p.

(b) Compute the tangent vectors to these two curves at the point p.

(c) Use these tangent vectors to compute a normal vector $\mathbf{n}$ to the graph $z = F(x, y)$ at the point p, and use this to give a linear equation for the tangent plane to the graph at p.

(d) Compute the gradient $\nabla G(x, y, z)$ of the function

$$G(x, y, z) = z - F(x, y) = z - x^2 + xy^3 - 1.$$

(e) Compute $\nabla G(1, 1, 1)$. How is it related to $\mathbf{n}$ geometrically?

(f) Use ∇G to give a linear approximation of $G(1.1, 0.9, 1.2)$.

(g) Consider a point $(1 + h_1, 1 + h_2, 1 + h_3)$. For which vectors $\langle h_1, h_2, h_3 \rangle$ is the linear approximation of $G(1 + h_1, 1 + h_2, 1 + h_3)$ equal to $G(1, 1, 1)$? Explain geometrically.

(2) Calculate the gradient and the total differential of the given function. Then, if indicated, evaluate at a point.

(a) $h(x, t) = e^{-3t} \sin(\pi x + 5t)$

(b) $g(x, y) = 2^x \sec(x^2 y)$

(c) $F(m, r) = Gm/r^2$, at $p = (100, 10)$

(3) Let $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.

(a) Use the gradient of f and the value $f(P)$ to obtain a linear approximation to $f(Q)$, where $P = (3, 4, 12)$ and $Q = (3.03, 3.96, 12.05)$.

(b) Use the differential df of $f(x, y, z)$ and the value $f(P)$ to approximate $f(Q)$ given $P(3, 4, 12)$ and $Q(3.03, 3.96, 12.05)$.

(4) Let $z = h(x, y) = 3x^2 - 9x + 4 + 2y^3 - 12y$.

(a) Find all points (x, y, z) for which $\dfrac{\partial h}{\partial x} = \dfrac{\partial h}{\partial y} = 0$.

(b) Through each of the points found in part (a), write parametric equations for the x-curves, y-curves, and tangent lines there.

(c) Compute unit vectors normal to each pair of tangent lines you found in part b) (at the appropriate points). What does this say geometrically about the tangent planes at those points? Does this have any implications for the surface?

(5) Parametrize the circle C_R of radius R centered at (P, Q) in the xy-plane. Find a parametric description of the tangent line through any point on C_R. What is the angle between the tangent line to a point on C_R and the diameter passing through that point? Justify your answer.

(6) The period of oscillation of a simple pendulum of length L in the presence of gravity g is

$$T = 2\pi\sqrt{L/g}.$$

Estimate the change in period if the length is changed from 2ft. to 2ft. 1in. and (simultaneously) g is changed from 32 ft/s^2 to 32.2 ft/s^2.