Math 241 §BL1

Problem Set 12

(1) Compute the gradients of the following functions.

(a) $f(x,y) = \sqrt{9 - x^2 - y^2}$

(b) $f(x,y) = \tan^{-1}(y/x)$.

(2) Let $\vec{u}$ and $\vec{v}$ be unit vectors in the plane and c a fixed real number. For differentiable functions $f(x,y)$ and $g(x,y)$ show

(a) $D_{\vec{u}+\vec{v}}f = D_{\vec{u}}f + D_{\vec{v}}f$,

(b) $D_{c\vec{u}}f = cD_{\vec{u}}f$, and

(c) $D_u(fg) = (D_u f)g + f(D_u g)$.

(3) In which unit direction does the function $f(x,y,z) = ze^{xy}$ increase fastest at the point $P(0,2,1)$?

(4) Find an equation for the tangent plane $\mathcal{T}$ at any point (a,b,c) on the surface $z = x^2 + y^2$. Write an equation for the line formed by the intersection of $\mathcal{T}$ and the xy-plane. Show that this line is tangent to the circle in the xy-plane centered at the origin with radius $\sqrt{(a^2+b^2)/4}$.

(5) Show that $D_{\vec{v}}f(x,y) = \pm\dfrac{\partial f}{\partial x}(x,y)$ for a vector $\vec{v}$ parallel to $\vec{i}$.

(6) The directional derivative of a function of two variables is (generally) again a function of two variables...ready to be differentiated in another direction. In particular, for two unit vectors $\vec{u}$ and $\vec{v}$, we define

$$D^2_{\vec{u}\vec{v}}f = D_{\vec{u}}(D_{\vec{v}}f).$$

Compute $D^2_{\vec{u}\vec{v}}f$ and $D^2_{\vec{v}\vec{u}}f$ where $f(x,y) = x^2y$, $\vec{u} = \langle -1, 1\rangle$ and $\vec{v} = \langle 2, 1\rangle$. Notice anything? As a challenge, try to show your observation holds for arbitrary choices of unit vectors and functions.

(7) Suppose $f(x, y)$ is a function with continuous second partial derivatives. Let $\nabla f(x, y) = \langle P(x, y), Q(x, y)\rangle$. What must be true about the derivatives of $P(x, y)$ and $Q(x, y)$?