Math 241 §BL1

Problem Set 15

(1) Find and classify all critical points of the following functions

(a) $f(x,y) = 6xy^2 - 2y^3 - 3y^4$

(b) $f(x,y) = e^{4y-x^2-y^2}$

(c) $f(x,y) = y\sqrt{x} - y^2 - x + 6y$

(2) Consider a rectangular box with three sides in the coordinate planes having their common vertex at the origin. Suppose that the vertex opposite the origin lies on the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

where a, b, and c are positive real numbers. What is the maximum possible volume of the box?

(3) Find the maximum and minimum values of $f(x,y) = x^2 + 2y^2$ on the disk bounded by the circle centered at the origin with radius 1.

(4) Find the point $P(x,y,z)$ on the surface described by $x^2y^2z = 4$ closest to the origin.

(5) Let $f(x,y) = x^3 - 3xy^2$. Show that the origin is the only critical point of $f(x,y)$ and that the discriminant (that is, the determinant of the Hessian) vanishes there. Now, choose lines through the origin in the plane (i.e. set $y = mx$ for various choices of m) and examine the behavior of $f(x,y)$ along these lines. Use this information to sketch the surface $z = f(x,y)$ near the origin.