Math 241 §BL1

Problem Set 17

(1) Use the method of Lagrange multipliers in the following problems.

(a) Maximize x^2y on the unit circle.

(b) Find all points on the hyperbola $xy = 1$ that lie closest to the origin.

(c) Find all points on the surface $xyz = 1$ closest to the origin.

(d) Minimize $f(x, y, z) = x^2 + y^2 + z^2$ on the line of intersection of the planes $x + 2y + 3z = 0$ and $2x + 3y + z = -4$.

(e) The plane $2y + 4z - 5 = 0$ meets the cone $z^2 = 4(x^2 + y^2)$ in a single curve. Find the point on this curve closest to the origin.

(2) Find the maximum value of $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the surfaces $x - y + z = 1$ and $x^2 + y^2 = 1$.

(3) Let p be a fixed real number. Prove that the rectangle with perimeter p that maximizes area is a square. Similarly, prove that the triangle with perimeter p that maximizes area is equilateral. You may find Heron's Formula helpful...

(4) Consider the graph (in the xy-plane) of the equation

$$x^2 + xy + y^2 = 3.$$

(a) Identify the graph of this equation by rotating about $\pi/4$ radians, i.e. let

$$\begin{aligned} x &= X\cos(\pi/4) - Y\sin(\pi/4) \\ y &= X\sin(\pi/4) + Y\cos(\pi/4). \end{aligned}$$

(b) Find the points on the above graph closest to and farthest from the origin using the method of Lagrange mulitpliers. (Hint: the system of equations

$$ax + by = 0$$
$$cx + dy = 0$$

has a nontrivial solution if any only if $ad - bc = 0$. Use this to find the Lagrange multiplier).