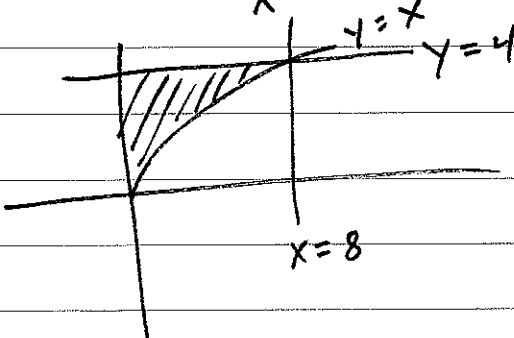


# PS 19 Selected Answers/Partial Solutions

$$\begin{aligned} \textcircled{3} \text{(i)} \int_0^1 \int_{y^{1/3}}^1 \frac{1}{\sqrt{1+x^2}} dx dy &= \int_0^1 \int_0^{x^3} \frac{1}{\sqrt{1+x^2}} dy dx \\ &= \int_0^1 \frac{x^3}{\sqrt{1+x^2}} dx \quad (*) \end{aligned}$$

Let  $u = 1+x^2$ ,  
then  $du = 2x dx$  so

$$\begin{aligned} (*) &= \frac{1}{2} \int_1^2 \frac{u-1}{\sqrt{u}} du \\ &= -\frac{1}{3}\sqrt{2} + \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \int_0^8 \int_{x^{2/3}}^4 x \cos(y^4) dy dx &= \int_0^4 \int_0^{\sqrt{y^3}} x \cos(y^4) dx dy \\ &= \int_0^4 \frac{y^3}{2} \cos(y^4) dy \\ &= \frac{1}{8} \int_0^{256} \cos(u) du \\ &= \frac{1}{8} \sin(256). \end{aligned}$$


PS 19 cont.

$$(iii) \int_0^4 \int_{\sqrt{y}}^2 \frac{ye^{x^2}}{x^3} dx dy = \int_0^2 \int_0^{x^2} \frac{ye^{x^2}}{x^3} dy dx$$

$$= \frac{1}{2} \int_0^2 xe^{x^2} dx$$

$$= \frac{1}{4} \int_0^4 e^u du$$

$$= \frac{e^4}{4} - \frac{1}{4}$$