

Math 241 §BL1

Problem Set 21

- (1) A lamina occupies the region bounded by the line $y = x + 2$ and the curve $y = x^2$. Find the center of mass of the lamina if its density at any point is proportional to the distance from the point and the y -axis.
- (2) A lamina occupies the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant. Suppose that at any point on the lamina, the density there is proportional to the distance from the point to the x -axis. What is the center of mass of the lamina?
- (3) The **moment of inertia about the origin** (also called the *polar moment*) is the sum $I_0 = I_x + I_y$.
- (a) Write a general formula for I_0 given a plane lamina bounding a region \mathcal{R} with density $\rho(x, y)$.
- (b) Find I_0 for the lamina that occupies the region in the plane bounded by $y = x^2/4$ and $y = 1$ having constant density.
- (c) A uniform (constant density) rectangular lamina with length a and height b and mass m has its center of mass at the origin. Show that

$$I_0 = \frac{1}{12}m(a^2 + b^2)$$