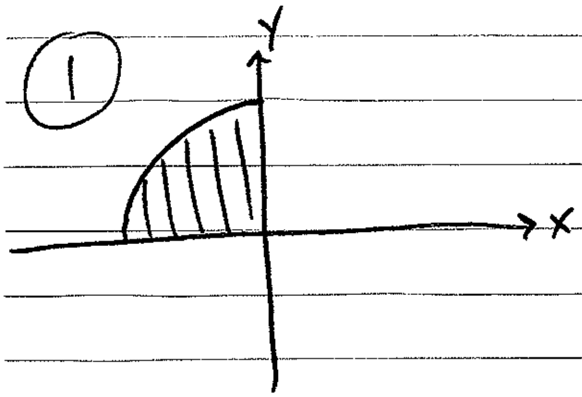
PS 23 Selected Answers/Partial Solutions

(1)



$$\int_{-2}^{0}\int_{0}^{\sqrt{4-x^2}} x^2+y^2 \, dy\,dx$$

$$= \int_{\pi/2}^{\pi}\int_{0}^{2} r^3 \, dr\,d\theta$$

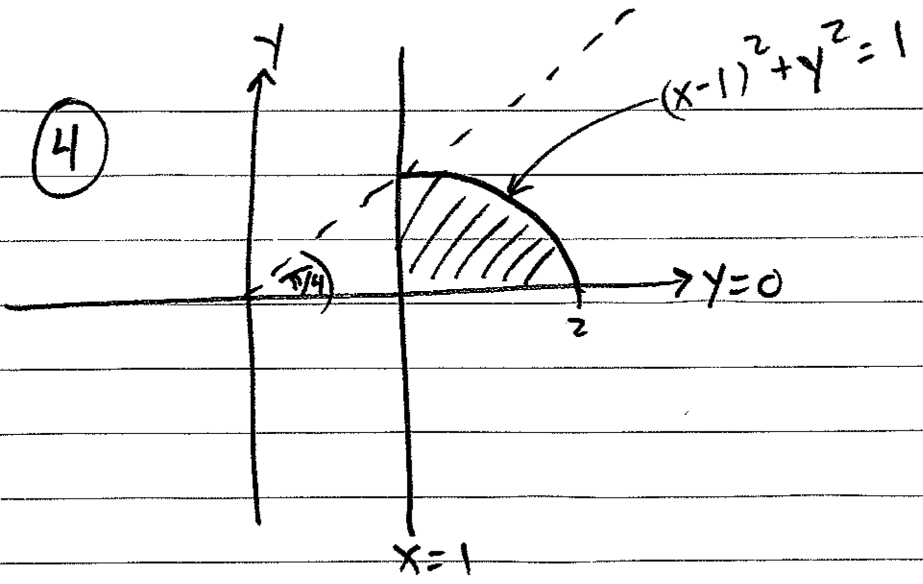
(2) $$\int_{0}^{-3}\int_{0}^{\sqrt{9-x^2}} 1 \, dy\,dx$$

(3) Let $I_m = \int_{0}^{\pi/2}\int_{0}^{m} \frac{r}{(1+r^2)^2} \, dr\,d\theta$.

Then,

$$\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{(1+x^2+y^2)^2} \, dx\,dy = \underbrace{\lim_{m\to\infty} I_m}_{= \lim_{m\to\infty} \frac{\pi}{4}\left(1 - \frac{1}{1+a^2}\right)} = \frac{\pi}{4}$$

(4)



Let $x = r\cos\theta$, $y = r\sin\theta$. Then,

$x = 1 \Rightarrow r = \frac{1}{\cos\theta}$ and

$(x-1)^2 + y^2 = 1 \Rightarrow r = 2\cos\theta$. So,

$$m = \iint_R \rho \, dA = \rho \int_0^{\pi/4} \int_{1/\cos\theta}^{2\cos\theta} r \, dr \, d\theta$$

$$= \rho \int_0^{\pi/4} \frac{r^2}{2} \Big|_{\frac{1}{\cos\theta}}^{2\cos\theta} d\theta$$

$$= \rho \int_0^{\pi/4} 2\cos^2\theta - \frac{1}{2}\sec^2\theta \, d\theta$$

$$= \frac{\pi}{4}\rho$$

(5) In (r,θ,z) coordinates, these surfaces can be described by

$$z^2 = 80 - 4r^2 \quad \text{(ellipsoid)}$$

$$z = 2r^2 \quad \text{(paraboloid)}$$

The curve of intersection of these surfaces lies above the curve

$$r^4 + r^2 - 20 = 0 \quad (*)$$

in the plane. The only real, positive solution of $(*)$ is $r = 2$.

So, an integral calculating the volume of the solid is

$$\iint_R z_{top} - z_{bot} \, dA$$

$$= \int_0^{2\pi} \int_0^2 \left(\sqrt{80-4r^2} - 2r^2\right) r \, dr \, d\theta$$

$$= 2\pi\left(\frac{80\sqrt{5} - 152}{3}\right)$$