

Math 241 §BL1

Problem Set 26

- (1) Consider the integral

$$\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} dz \, dy \, dx.$$

- (a) What does this integral evaluate for us?  
(b) Sketch the region of integration.  
(c) Rewrite the integral in the following orders:  $dx \, dy \, dz$  and  $dz \, dx \, dy$ .  
(d) Evaluate the integral.
- (2) Find the mass of the upper unit half ball in  $\mathbb{R}^3$  with density  $\rho = z$ .
- (3) The average value of a function  $f(x, y, z)$  over a solid region  $T$  is given by

$$\bar{f} = \frac{1}{\text{Vol}(T)} \iiint_T f(x, y, z) \, dV$$

where  $\text{Vol}(T)$  denotes the volume of the solid. Find the average value of  $f(x, y, z) = x^2z + y^2z$  over the region enclosed by  $z = 1 - x^2 - y^2$  and  $z = 0$ .

- (4) Sketch the surface described by the cylindrical equation  $z = r$ . Compute the volume of the solid bounded by this surface and the planes  $z = \pm 2$ .
- (5) Use cylindrical coordinates to find the volume of the solid bounded by the surfaces  $z = 2x^2 + 2y^2$  and  $z = 48 - x^2 - y^2$ .
- (6) Find the volume of the region  $T$  bounded by the surfaces  $z = x^2 + y^2$  and  $z = 36 - 3x^2 - 3y^2$ .
- (7) Find the volume inside both the cylinder  $x^2 + y^2 = 1$  and the sphere  $x^2 + y^2 + z^2 = 4$ .
- (8) Use a triple integral and an appropriate change of coordinates to calculate the volume of the region bounded by the  $xy$ -plane, the surface  $z = x^2 + y^2$  and the

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cylinder over

$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$