

Math 241 §BL1

Problem Set 27

(1) Suppose that $f(x)$ and $g(y)$ are continuous functions. Show that $\vec{F} = \langle f(x), g(y) \rangle$ is conservative.

(2) Show that

$$\vec{F}(x, y) = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle$$

is not conservative on the entire plane (Hint: see Problem Set 12).

(3) Consider the vector field $\vec{F}(x, y) = \langle y, x \rangle$.

(a) Show that \vec{F} is conservative by finding a potential function $f(x, y)$.

(b) Show that for any real c , the curve defined by $f(x, y) = c$ intersects flow lines of \vec{F} at right angles (it's not necessary to find the flow lines explicitly in order to do this).

(c) Let $g(s) = c_1 e^s + c_2 e^{-s}$, where c_1 and c_2 are constants. Show that $g''(s) = g(s)$. It is a fact that every solution of the latter differential equation is of the same form as $g(s)$.

(d) Find a general formula for the flow lines of \vec{F} .

(e) Suppose a flow line of \vec{F} passes through $(1, 1)$ at a certain time s_0 . Conclude as in (b) that the level curve of f passing through $(1, 1)$ is perpendicular to the flow line at that point using the formula you found in (d).

(4) Consider the vector field

$$\vec{F}(x, y) = \vec{i} + x\vec{j}.$$

on the plane. A particle is dropped onto the plane at time $t = 0$ and at position $(-2, 2)$.

(a) Describe the motion of the particle.

- (b) How far is the particle from its initial position at time $t = 4$?
- (c) Estimate the distance that the particle has traveled at time $t = 4$. Hint:

$$\frac{d}{du} \left(\frac{u}{2} \sqrt{1+u^2} + \frac{1}{2} \ln(u + \sqrt{u^2+1}) \right) = \sqrt{1+u^2}$$