

Math 241 §BL1

Problem Set 30

(1) Determine whether or not the following vector fields are conservative. For vector fields that are conservative, find potential functions.

(a) $\vec{F} = \langle 2y + \sin 2x, 3x + \cos 3y \rangle$

(b) $\vec{F} = \langle x + \arctan y, (x + y)/(1 + y^2) \rangle$

(c) $\vec{F} = e^{x-y} \langle xy + y, xy + x \rangle$

(d) $\vec{F} = \langle e^x \sin y + \tan y, e^x \cos y + x \sec^2 y \rangle$

(2) Evaluate the line integral

$$\int_C xyz \, ds$$

where C is the path from $(1, 1, 2)$ to $(2, 3, 6)$ consisting of the three line segments parallel first to the x -axis, then the y -axis, and finally the z -axis.

(3) Evaluate the line integral

$$\int_C xe^y \, dx$$

where C is the portion of the curve $x = e^y$ from $(1, 0)$ to $(e, 1)$.

(4) Evaluate the line integral

$$\int_C xy \, ds$$

where C is the portion of the elliptical helix parameterized by $\vec{r}(t) = \langle 4 \cos t, 9 \sin t, 7t \rangle$, $0 \leq t \leq 5\pi/2$.

(5) The average distance \bar{D} from a fixed point P to a point (x, y) on a planar curve C is given by

$$\bar{D} = \frac{1}{s} \int_C D(x, y) \, ds$$

where $D(x, y)$ is the distance from P to (x, y) and s is the arclength of s .

Consider the spiral parameterized by $\vec{r}(t) = \langle e^{-t} \cos t, e^{-t} \sin t \rangle$ starting at $(1, 0)$ and approaching the origin as $t \rightarrow \infty$. Find the average distance from the origin to points on the spiral.

(6) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = \langle e^{x-1}, xy \rangle$ and $\vec{r}(t) = \langle t^2, t^3 \rangle$, $0 \leq t \leq 1$.

(7) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = \langle z, y, -x \rangle$ and $\vec{r}(t) = \langle t, \sin t, \cos t \rangle$, $0 \leq t \leq \pi$.

(8) Let $\vec{F}_1 = \langle x, y \rangle$ and $\vec{F}_2 = \langle -y, x \rangle$. Compute the work done by these vector fields on a particle that moves once counterclockwise around the circle of radius R .

(9) An electric charge at the origin exerts a force on a charged particle at position $\vec{r} = \langle x, y, z \rangle$ given by

$$\vec{F} = \frac{K\vec{r}}{\|\vec{r}\|^3}$$

where K is a constant. Find the work done by \vec{F} on the particle as it moves in a straight path from $(2, 0, 0)$ to $(2, 1, 5)$.

(10) An electric current I in a straight wire generates a magnetic field \vec{B} in the space around the wire. It is a fact that \vec{B} is tangent to any circle C centered around the wire lying in a plane perpendicular to the wire. By Ampere's Law

$$\int_C \vec{B} \cdot d\vec{r} = \mu I$$

where μ is constant. Show that the magnitude of the field \vec{B} is proportional to I and inversely proportional to the distance from the wire.