Math 241 §BL1

Problem Set 37

(1) Evaluate the surface integral

$$\iint_\Sigma y \, dS$$

where Σ is the surface $z = x + y^2$, $0 \leq x \leq 1$, $0 \leq y \leq 2$.

(2) Compute

$$\iint_\Sigma x^2 \, dS$$

where Σ is the unit sphere in $\mathbb{R}^3$.

(3) Consider the surface integral

$$\iint_\Sigma z \, dS$$

where Σ is the surface with sides S_1 given by the cylinder $x^2 + y^2 = 1$, S_2 given by the unit disk in the xy-plane, and S_3 given by the plane $z = x + 1$. Evaluate this integral as follows:

(a) Parameterize S_1 using (θ, z) coordinates.

(b) Evaluate the integral over the surface S_2 without parameterizing.

(c) Parameterize S_3 in (Des)cartesian coordinates and evaluate the resulting integral using polar coordinates.

(4) (i) Suppose a surface Σ is given by $z = f(x, y)$, where f has domain D. Show that the flux of a vector field $\vec{F} = \langle P, Q, R \rangle$ across Σ is given by

$$\iint_D \left(-P\frac{\partial f}{\partial x} - Q\frac{\partial f}{\partial y} + R \right) \, dA$$

(ii) Compute the flux of $\vec{F} = y\vec{i} + x\vec{j} + z\vec{k}$ across the closed surface Σ with "top" $z = 1 - x^2 - y^2$ and "bottom" $z = 0$. (Hint: use part (i) for the top, and take $\vec{n} = -\vec{k}$ for the bottom).

(5) Find the flux of the vector field $\vec{F} = \langle x, y, z^4 \rangle$ across the surface Σ that is part of the cone $z = \sqrt{x^2 + y^2}$ under $z = 1$ with downward unit normal.

(6) Find the flux of $\vec{F} = \langle x, -z, y \rangle$ across the part of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant with orientation toward the origin.

(7) Let $\vec{F} = \langle z, y, x \rangle$ and S^2 be the unit sphere.

(i) Parameterize S^2 via some $\vec{r}(\phi, \theta)$.

(ii) Compute $\vec{F}(\vec{r}(\phi, \theta))$.

(iii) Compute $\vec{r}_\phi \times \vec{r}_\theta$.

(iv) Compute

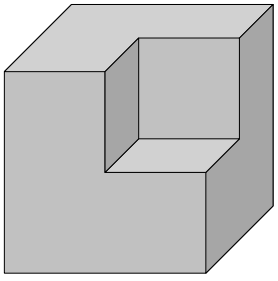
$$\iint_{S^2} \vec{F} \cdot \vec{n} \, dS$$

(v) Compute

$$\iiint_{B^3} \text{div } \vec{F} \, dV$$

where B^3 is the unit ball bounded by S^2. Compare your answer with (iv).

(8) Let $\vec{F} = \langle x, y, z \rangle$. Consider the outward oriented surface whose boundary is formed by taking the cube of side length 2 in the first octant and removing the unit cube with vertex at $(2, 2, 2)$.



Calculate the flux of $\vec{F}$ across the surface.