

Math 241 §BL1

Problem Set 38

(1) Show that if $\vec{F}(x, y, z)$ is a vector field whose components have continuous second-order partial derivatives, then $\nabla \cdot (\nabla \times \vec{F}) = 0$. Use this to solve Problem (1) on Problem Set 35.

(2) Find the flux of $\vec{F} = \langle xy \sin z, \cos(xz), y \cos z \rangle$ across the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

(3) Use the Divergence Theorem to solve problem 8 on Problem Set 37.

(4) Use the Divergence Theorem to evaluate

$$\iint_{\Sigma} (2x + 2y + z^2) dS$$

where Σ is the unit sphere in \mathbb{R}^3 .

(5) Let $\vec{F} = \langle 3y, -2x, xyz \rangle$ and Σ be the surface described by $z = \sqrt{4 - x^2 - y^2}$.

(a) Compute $\iint_{\Sigma} \text{curl } F \cdot \vec{n} dS$

(b) Compute $\int_C \vec{F} \cdot d\vec{r}$ where C is the circle having radius 2 centered at $(0, 0)$ in the xy -plane.

(c) Compare your answers to parts (a) and (b).