Math 241 §BL1, Suggested Practice Problems 2 (Corrected 2/27/08, 1:49 p.m.)

These problems are designed to give you practice on some basic skills. You should *not* conclude that they test everything you will need to understand for the exam!

(1) Find parametric descriptions of tangent lines to the x and y-curves on the surface $z = h(x,y) = 1 - 3x^2 - 4y^2$ at the point $P = (1,1,-6)$. Compute the tangent plane to the surface at that point.

(2) Find the gradient of the function $g(x,y,z) = 2^y \sin(x^2 z) - zy\cos z$ at the point $P = (1,0,\pi)$.

(3) Find the directional derivative of $f(x,y) = \tan^{-1}(y/x)$ in the direction of $\vec{v} = \langle 3,4\rangle$ at the point $P = (-3,3)$. [Note: recall that $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$.]

(4) In which unit direction does $f(x,y) = \ln(x^2+y^2)$ increase most rapidly at the point $P = (3,4)$?

(5) Suppose $z = z(x,y)$ is a function of x and y that satisfies $xe^{xy} + ye^{zx} + ze^{xy} = 0$. Find $\frac{\partial z}{\partial x}(0,0)$.

(6) Let $f(x,y) = xy - x + 2x^3$, $x(s,t) = 2t - st$, $y(s,t) = 3s^2$. Use the Chain Rule to compute $\frac{\partial F}{\partial s}(2,1)$ where $F(s,t) = f\big(x(s,t), y(s,t)\big)$.

(7) Find and classify all critical points of the function $f(x,y) = x^4 + y^4 - 4xy + 1$.

(8) Compute an equation of the tangent plane to the surface defined by $x^2yz + 2y^3z - x = 2$ at the point $P = (1,1,1)$.

(9) **Use Lagrange Multipliers** to find the maximum and minimum values of $f(x,y) = x^2 + 2y^2$ on the unit circle in the xy-plane.

(10) Find all points on the sphere of radius 2 centered at the origin closest to and farthest from the point $P = (3,1,-1)$ (hint: minimize the square of the distance and find the value of the Lagrange multiplier).

Solutions.

(1) Tangent line to x-curve $\ell(t) = \langle 1+t, 1, -6-6t\rangle$. Tangent line to y-curve $m(t) = \langle 1, 1+t, -6-8t\rangle$. Tangent plane to surface $6x + 8y + z = 7$.

(2) $\nabla g(1, 0, \pi) = \langle -2\pi, \pi, -1\rangle$.

(3) $-7/30$

(4) $\langle 3/5, 4/5\rangle$

(5) $\dfrac{\partial z}{\partial x}(0,0) = -1$

(6) -11

(7) $(0,0)$ is a saddle point, $(1,1)$ is a local min, $(-1,-1)$ is a local min

(8) $x + 7y + 3z = 11$

(9) minimum value 1 attained at $(1,0)$ and $(-1,0)$, maximum value 2 attained at $(0,1)$ and $(0,-1)$ [Remark: Once you have used Lagrange multipliers to solve the problem, there is a simple way to check that you have the right solution.]

(10) Closest is $(6/\sqrt{11}, 2/\sqrt{11}, -2/\sqrt{11})$, farthest is $(-6/\sqrt{11}, -2/\sqrt{11}, 2/\sqrt{11})$