1. Let $K/F$ be an algebraic extension. Suppose $R$ is a subring contained in $K$ which contains $F$. Prove that $R$ is actually a subfield of $K$.

2. Prove that $\alpha = \cos(2\pi/5)$ is a constructable number. Use this to show that the regular 5-gon is constructable by straightedge and compass.

3. Find the splitting field $K$ of $x^4 - 2$ over $\mathbb{Q}$. What is $[K:\mathbb{Q}]$?

4. Find the splitting field $K$ of $x^4 + x^2 + 1$ over $\mathbb{Q}$. What is $[K:\mathbb{Q}]$?

5. Suppose $K/F$ is the splitting field for a polynomial $f(x) \in F[x]$. Let $g(x) \in F[x]$ be irreducible. Show that if $g$ has a root in $K$ then it splits completely in $K[x]$. 