Math 131 Homework #9; Due Tuesday April 8

Munkres: 9.51 2(b), 3.

9.52 1, 4, 5, 6.

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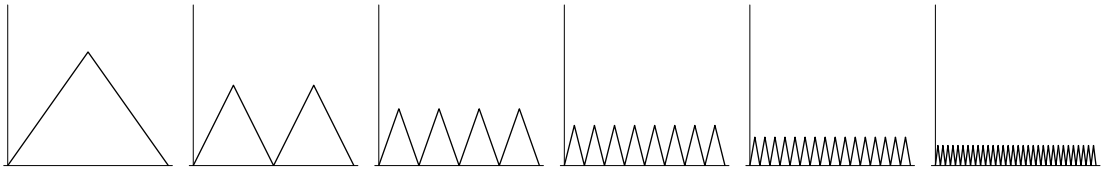
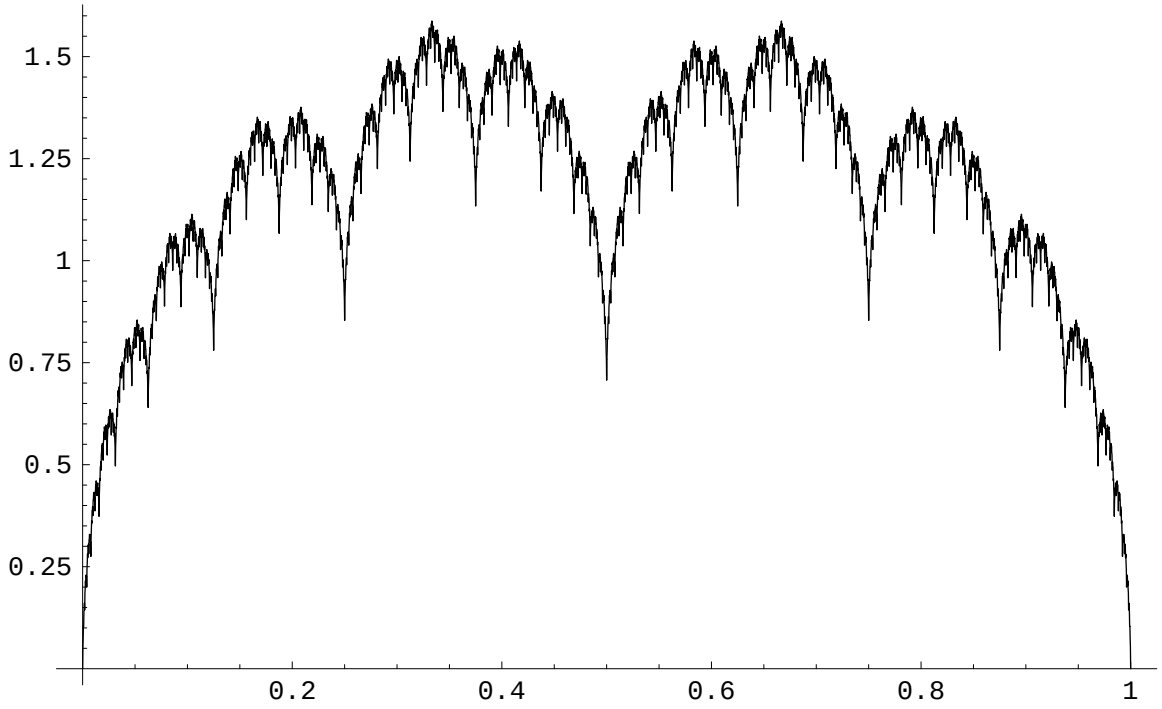
N1: Let $g_n : [0,1] \to \mathbb{R}$ the function which is defined as follows. The function g_n takes on the value 0 at points of the form $k/2^{n-1} \in [0,1]$ for k and integer, and the value $2^{-n/2}$ at points of the form $k/2^n$ for k odd. In between these points, the graph of g_n is a straight line. In other words, the graph of g_n is a mountain range picture with 2^{n-1} peaks of height $2^{-n/2}$. Below are the graphs of $g_1, \ldots g_6$.



Show that the function $f = \sum_{i=1}^{\infty} g_n$ exists as a function on $[0,1]$, is continuous, and is nowhere differentiable. An appoximation to the graph of f is below.



Hint: If f is differentiable at x prove that

$$\sup_{y \in [0,1] \setminus \{x\}} \frac{|f(x) - f(y)|}{|x - y|}$$

is finite.