Midterm for Math 20

Friday, October 22, 1999

There are 100 points possible on this exam. Take care to note that problems are not weighted equally. Calculators, books, notes and suchlike aids to gracious living are *not* allowed. On the questions that ask "Why?" a long answer is not what I'm looking for – 1-2 sentances is what I want. You have 55 minutes. Show your work. Good luck!

1. Consider the vectors $\mathbf{v} = (1, 2, -1)$ and $\mathbf{w} = (1, 0, 1)$ in $\mathbb{R}^3$. Compute
   (a) $\mathbf{v} - 2\mathbf{w}$ **(4 points)**
   (b) The angle between $\mathbf{v}$ and $\mathbf{w}$. **(4 points)**

2. Consider the matrices

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix}, D = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 1 & 1 & -2 \end{pmatrix}.$$

   Compute the following, when possible (when not possible, just answer "not possible") **(8 points each)**
   (a) AB.
   (b) BA.
   (c) $A^T + 2B$.
   (d) $\det(D)$.

3. Let P_1 be the plane in $\mathbb{R}^3$ defined by $x + y = 1$, and P_2 the plane defined by $2x + y + 3z = 2$.
   (a) Find normal vectors for P_1 and P_2. **(5 points)**
   (b) These two planes are not parallel. Why? **(5 points)**
   (c) Parameterize the line which is the intersection of P_1 and P_2. **(10 points)**

4. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation with matrix $A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$, and $S\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation with matrix $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. **(5 points each)**
   (a) Draw what happens to the unit square with vertices $(0, 0), (1, 0), (0, 1)$, an $(1, 1)$ under the transformation T.
   (b) Based on your answer to (a), it the matrix A invertible or not? Why?
   (c) Find the matrix associated with the linear transformation $T \circ S$.

5. **(10 points)** Find the matrix of the linear transformation $T\colon\ \mathbb{R}^2 \to \mathbb{R}^2$ which acts as shown:

6. Consider the line segment L in $\mathbb{R}^3$ joining the points $P = (2, 0, -2)$ and $Q = (4, 2, 0)$.

   (a) The midpoint M of L is the point half-way between P and Q. Find M. **(5 points)**

   (b) Find the equation of the plane which intersects L in its midpoint M and which is orthogonal to L. **(10 points)**