I. A. Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, 2(x-3)^n}{n^4}$$

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{\dfrac{2|x-3|^{n+1}}{(n+1)^4}}{\dfrac{2|x-3|^n}{n^4}} = \lim_{n\to\infty} |x-3| \frac{n^4}{(n+1)^4}$$

$$= |x-3|$$

- Converges if $|x-3|<1$
  or $2<x<4$
- Diverges if $|x-3|>1$
  or $x<2$ or $x>4$

- If $x=2$ $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{2n+1}\cdot 2}{n^4}$

$$= -2\sum_{n=1}^{\infty}\frac{1}{n^4}$$

Convergent p-series

- If $x=4$ see below

IOC = $[2,4]$

B. Give the reason why the series does or does not converge at the *largest* endpoint of its interval of convergence.

When $x=4$ the series is

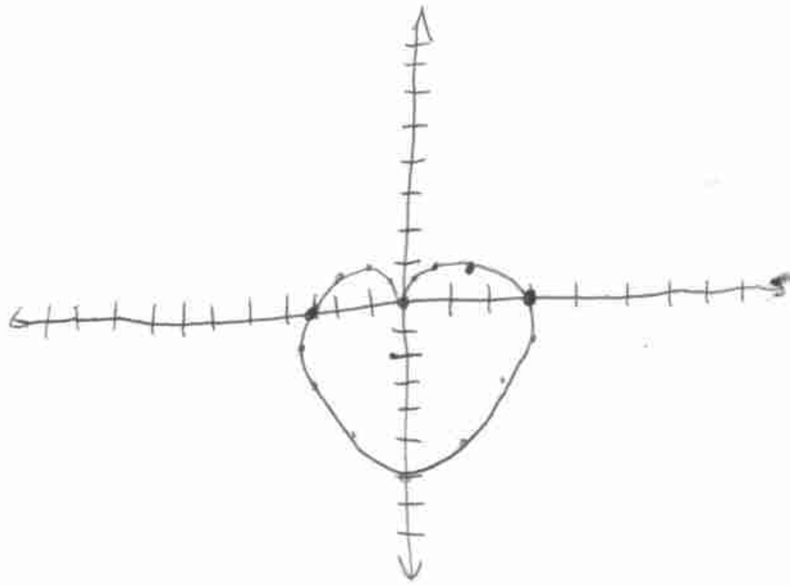
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, 2\cdot 1^n}{n^4} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\cdot 2}{n^4}$$

which is an alternating series whose terms go to 0 and the absolute value of the terms decreases, so the series converges by Alternating Series Test

II.A. Sketch the polar curve defined by the equation

$$r = 3 - 3\sin(\theta).$$

θ	r
0	3
$\pi/6$	$3/2$
$\pi/4$	$3 - 3\sqrt{2}/2$
$\pi/3$	$3 - 3\sqrt{3}/2$
$\pi/2$	0
$2\pi/3$	$3 - 3\sqrt{3}/2$
$3\pi/4$	$3 - 3\sqrt{2}/2$
$5\pi/6$	$3/2$
π	3
$7\pi/6$	$9/2$
$5\pi/4$	$3 + 3\sqrt{2}/2$
$4\pi/3$	$3 + 3\sqrt{3}/2$
$3\pi/2$	6
$5\pi/3$	$3 + 3\sqrt{3}/2$
$7\pi/4$	$3 + 3\sqrt{2}/2$
$11\pi/6$	$9/2$



B. Write down the integral in polar coordinates giving the area of the region bounded by the curve of part A. Do *NOT* evaluate the integral.

$$\int_0^{2\pi} \frac{1}{2} r^2 \, d\theta = \int_0^{2\pi} \frac{1}{2}(3 - 3\sin\theta)^2 \, d\theta$$