Math 231/199: Calculus 2 Merit

Worksheet 1

August 23, 2007

All of trigonometry comes from six definitions (those of the six trig functions) and three formulas:

(1) $\sin^2(\theta) + \cos^2(\theta) = 1$

(2) $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$

(3) $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$.

A true reductionist would even say that these three formulas are all consequences of the Pythagorean Theorem, so that, in a way, there is only one trig formula: the Pythagorean Theorem itself.

To support the bold claim of the last paragraph, use the three formulas above to answer the following questions.

(1) Show that $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ and $\cos(2\theta) = 1 - 2\sin^2(\theta)$.

(2) Find formulas for $\sin^2(\theta)$ and $\cos^2(\theta)$ in terms of cosines only. (Hint: use something from Question 1)

(3) Find a formula for $\sin(4\theta)$ in terms of $\sin(\theta)$ and $\cos(\theta)$.

(4) Use Formulas 2 and 3 to show that $\sin^2(\alpha + \beta) + \cos^2(\alpha + \beta) = 1$.

(5) Show that $\tan^2(\theta) + 1 = \sec^2(\theta)$ and $\cot^2(\theta) + 1 = \csc^2(\theta)$.

(6) The Law of Cosines says that for a triangle with side lengths x, y, and z with an angle α opposite the side of length x,

$$x^2 = y^2 + z^2 - 2yz\cos(\alpha).$$

The Law of Cosines can be derived from the Pythagorean Theorem.

Using the following picture as a guide, use the Law of Cosines to derive Formula 3. (Hint: Start by writing an equation involving $\cos(\alpha+\beta)$. Try simplifying it with the Pythagorean Theorem. If necessary, write all lengths in terms of sines and cosines.)