Math 231/199: Calculus 2 Merit
Worksheet 10
October 9, 2007

(1) For which values of p does $\int_1^\infty 1/x^p \, \mathrm{d}x$ converge?

For which values of p does $\sum_{k=1}^\infty 1/k^p$ converge? Which test are you using?

(2) For which values of p does $\sum_{k=1}^\infty \ln(k)/k^p$ converge? (A Very Important Fact may be helpful).

(3) Determine if the following series converge or diverge. State which test you are using.

$$\sum_{k=1}^{\infty} k^{-9/10}$$

$$\sum_{k=1}^{\infty} k^{-11/10}$$

$$\sum_{k=1}^{\infty} \frac{4}{(2+4k)^2}$$

$$\sum_{k=1}^{\infty} \frac{1}{k\ln(k)}$$

$$\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2+1}$$

$$\sum_{k=1}^{\infty} \frac{e^{1/k}}{k^2}$$

$$\sum_{k=1}^{\infty} \frac{e^{-\sqrt{k}}}{\sqrt{k}}$$

$$\sum_{k=1}^{\infty} \frac{1}{\cos^2(k)}$$

$$\sum_{k=1}^{\infty} \frac{\ln(k)}{k}$$

$$\sum_{k=1}^{\infty} \frac{\ln(k)}{k^{5/4}}$$

$$\sum_{k=1}^{\infty} \frac{2+\cos(k)}{k}$$

(4) Use the Limit Comparison Test to determine if the following series converge or diverge:

$$\sum_{k=1}^{\infty} \frac{5k^3 + 6k^2 - 19k + 4}{k^4 + 3k + 2}$$

$$\sum_{k=1}^{\infty} \frac{k+1}{k+2}$$

(5) Explain why if $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ are series with $0 < a_k < b_k$ whenever k is large and $\sum_{k=1}^{\infty} b_k$ converges, then $\sum_{k=1}^{\infty} a_k$ converges.

(6) Show that if $\sum_{k=1}^{\infty} a_k$ is a convergent series with $a_k > 0$, then $\sum_{k=1}^{\infty} a_k^2$ also converges.

(7) Let $\sum_{k=1}^{\infty} a_k$ be a series with positive terms. Draw a picture of the following (superimposed) in such a way that $h(x) \leq f(x) \leq g(x)$ for all x: (1) a function $g(x)$ on the interval $[1, \infty)$, the area under which is $\sum_{k=1}^{\infty} a_k$ (2) a function $h(x)$ on $[1, \infty)$, the area under which is $\sum_{k=2}^{\infty} a_k$ (3) a positive, decreasing, continuous function $f(x)$ on the interval $[1, \infty)$ such that $f(k) = a_k$ for all integers k.

(8) Based on your picture from the last question, if $\int_1^\infty f(x)\,\mathrm{d}x$ diverges, what can you say about $\sum_{k=1}^\infty a_k$? What test are you using?

If $\int_1^\infty f(x)\,\mathrm{d}x$ converges, what can you say about $\sum_{k=1}^\infty a_k$? What test are you using?

In terms of the function f, what sort of upper bound can you put on $\sum_{k=n}^\infty a_k$?

Suppose the integral $\int_1^\infty f(x)\,\mathrm{d}x$ converges, so that $\sum_{k=1}^\infty a_k$ converges by this Integral Test. Suppose $\sum_{k=1}^\infty a_k = S$. If S_n is the nth partial sum, use your answer from the last part to put an upper bound on $|S - S_n|$.

(9) Show that the series $\sum_{k=1}^{\infty} k^{-4}$ converges. Estimate the error made in approximating the value of the series with S_{100}, the 100th partial sum.

(10) Show that the series $\sum_{k=1}^{\infty} ke^{-k^2}$ converges. Let S be the sum of this series. Find a value of n such that the nth partial sum S_n satisfies $|S - S_n| \leq .01$. What does this say about approximating S with S_n?

(11) Determine all values of p such that

$$\int_2^\infty \frac{1}{x(\ln(x))^p}\,\mathrm{d}x$$

converges.

(12) Suppose $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ are positive term series. Answer the following with "converges", "diverges", or "don't know":

(1) If $\sum_{k=1}^{\infty} a_k$ converges and $b_k \geq a_k$ for $k \geq 10$, then $\sum_{k=1}^{\infty} b_k \ldots$

(2) If $\sum_{k=1}^{\infty} a_k$ converges and $\lim_{k\to\infty} b_k/a_k = 0$, then $\sum_{k=1}^{\infty} b_k \ldots$

(3) If $\sum_{k=1}^{\infty} a_k$ converges and $b_k \leq a_k$ for $k \geq 10$, then $\sum_{k=1}^{\infty} b_k \ldots$

(4) If $\sum_{k=1}^{\infty} a_k$ converges and $\lim_{k\to\infty} b_k/a_k = \infty$, then $\sum_{k=1}^{\infty} b_k \ldots$

(5) If $\sum_{k=1}^{\infty} a_k$ diverges and $b_k \geq a_k$ for $k \geq 10$, then $\sum_{k=1}^{\infty} b_k \ldots$

(6) If $\sum_{k=1}^{\infty} a_k$ diverges and $\lim_{k\to\infty} b_k/a_k = 0$, then $\sum_{k=1}^{\infty} b_k \ldots$

(7) If $\sum_{k=1}^{\infty} a_k$ diverges and $b_k \leq a_k$ for $k \geq 10$, then $\sum_{k=1}^{\infty} b_k \ldots$

(8) If $\sum_{k=1}^{\infty} a_k$ diverges and $\lim_{k\to\infty} b_k/a_k = \infty$, then $\sum_{k=1}^{\infty} b_k \ldots$

(13) Determine if the series $1 + 1/3 + 1/5 + 1/7 + \ldots$ converges or diverges.

(14) Show that $\sum_{k=2}^{\infty} 1/\ln(k)^{\ln(k)}$ and $\sum_{k=2}^{\infty} 1/\ln(k)^k$ both converge but $\sum_{k=2}^{\infty} 1/\ln(k)^a$ diverges for any fixed integer a.