Math 231/199: Calculus 2 Merit

Worksheet 12

October 16, 2007

(1) When applied to the same series, the Ratio Test and Root Test always give the same value of ρ. Explain, then, why it would be a bad idea to test the convergence of a series with the Root Test if the Ratio Test is inconclusive.

(2) List all the tests you know which conclude that a series is conditionally convergent.

(3) Given your answer to the last question, how can you prove that a series converges conditionally?

(4) Simplify the following expressions:

$$\frac{(k+1)!}{k!}$$

$$\frac{(2k)!}{(2k+3)!}$$

$$\frac{1 \cdot 3 \cdot 5 \cdots (2n+1)}{1 \cdot 3 \cdot 5 \cdots (2n+5)}$$

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(5) Let a be any constant. Show that $\lim_{k\to\infty} \sqrt[k]{k^a} = 1$.(HINT: take the natural logarithm)

(6) Use the Root Test to determine if

$$\sum_{k=1}^{\infty} \frac{k^{17} 2^k}{3^k}$$

converges or diverges.

(7) Let $f(n)$ be a function where $f(n) = 1$ if n is divisible by 3 and $f(n) = 0$ otherwise. Show that $\sum_{k=1}^{\infty}(-1)^{f(k)}k^{-2}$ converges. In general, if $g(n)$ is any function which takes integer values, show that $\sum_{k=1}^{\infty}(-1)^{g(k)}k^{-2}$ converges.

(8) Show that $\sum_{k=1}^{\infty}\sin(k)/k^2$ converges.

(9) Determine if the following series converge or diverge:

$$\sum_{k=1}^{\infty} (-1)^k \frac{3}{k!}$$

$$\sum_{k=1}^{\infty} (-1)^k 2^k$$

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{k^2+1}$$

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2+1}{k}$$

$$\sum_{k=1}^{\infty}(-1)^k \frac{10^k}{k!}$$

$$\sum_{k=1}^{\infty}\left(\frac{1-3k}{4k}\right)^k$$

$$\sum_{k=1}^{\infty}\left(\frac{e^k}{k^2}\right)^k$$

$$\sum_{k=1}^{\infty}\frac{\cos(k)}{k^3}$$

$$\sum_{k=1}^{\infty} \frac{e^{3k}}{k^{3k}}$$

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{\sqrt{k}}{k+1}$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{\ln(k+3)}$$

$$\sum_{k=1}^{\infty} \frac{3}{k^k}$$

(10) Suppose you have two series $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ whose terms are identical except that all the a_k's are positive whereas the b_k's may be positive or negative. Which of these series is more likely to converge? What does this say about absolutely convergent series?

(11) Show that the series $\sum_{k=1}^{\infty} k!/k^k$ converges. The fact that

$$\lim_{k\to\infty} \left(1 + \frac{1}{k}\right)^k = e$$

may be helpful.

(12) Determine whether

$$\sum_{k=1}^{\infty} \frac{k!}{1 \cdot 3 \cdot 5 \cdots (2k-1)}$$

converges or diverges.

(13) Find all values of p such that $\sum_{k=1}^{\infty} p^k/k$ converges.

(14) Find all values of p such that $\sum_{k=1}^{\infty} p^k/k^2$ converges.

(15) Based on your experiences, for what kinds of series is the Root Test easier to use than the Ratio Test?

(16) What is your favorite number?

Consider the following procedure. Start with the alternating harmonic series $\sum_{k=1}^{\infty}(-1)^k/k$. Take only positive terms of the alternating harmonic series until the sum of the terms you chose first exceeds your favorite number. Then take negative terms until the sum is less than your favorite number. Continue in this fashion, taking some of the remaining positive terms until the sum is above your favorite number and then taking negative terms until the sum is below your favorite number. Explain why this procedure will produce an ordering of the terms of the alternating harmonic series which converges to your favorite number. Doesn't this mean that if you rearrange the terms in a series, you can get a different value?