Math 231/199: Calculus 2 Merit

Worksheet 13

October 18, 2007

(1) Given that the power series representation of $1/(1-x)$ is $\sum_{k=0}^{\infty} x^k$, find the power series representation of the following functions.

$$\frac{1}{1+x}$$

$$\frac{1}{1-2x}$$

$$\frac{1}{1+x^2}$$

(2) Find the interval of convergence of the following power series represent as well as the functions they represent.

$$\sum_{k=0}^{\infty}(x-42)^k$$

$$\sum_{k=1}^{\infty} x^k$$

$$\sum_{k=0}^{\infty}(-1)^k(x/2)^k$$

$$\sum_{k=0}^{\infty} 25(x/3)^k$$

(3) Find the interval of convergence and radius of convergence of the following power series.

$$\sum_{k=1}^{\infty} \frac{11^k}{k!}\, x^k$$

$$\sum_{k=1}^{\infty} \frac{(x-2)^k}{k!}$$

$$\sum_{k=1}^{\infty} \frac{2^k (x+2)^k}{k^2}$$

$$\sum_{k=1}^{\infty} \frac{7^k x^k}{\sqrt{k}}$$

$$\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}\, x^k$$

$$\sum_{k=1}^{\infty} (-1)^k k^2 (x+17)^k$$

(4) Find the power series representations of the following functions.

$$f(x) = 3\tan^{-1}(x)$$

$$g(x) = \ln(1 + x^2)$$

$$h(x) = 1/(x - 1)^2$$

(5) Suppose the series $\sum_{k=1}^{\infty} a_k x^k$ has radius of convergence r. What is the radius of convergence of $\sum_{k=1}^{\infty} a_k (x-c)^k$?

(6) Start with the equation $x/(1-x) + x/(x-1) = 0$. Rewrite it as $x/(1-x) + 1/(1-1/x) = 0$. Find the power series representation for the left-hand side by combining the power series for both parts. This power series should always equal 0; does it? What went wrong?

(7) Find a power series representation for $\sum_{k=1}^{\infty} kx^k$. Determine its interval of convergence. Use your answer to determine $\sum_{k=1}^{\infty} k/5^k$.